

Volumes of Spheres in n-dimensional Space

Here are formulas for the volume of spheres in n -dimensional space. V_n is the volume of the unit sphere in n -dimensional space. To get $V_n(a)$, the volume of the sphere of radius a , you use

$$V_n(a) = V_n a^n$$

The general formula for for V_n is

$$V_n = \pi \frac{\Gamma(\frac{1}{2})^{n-2}}{\Gamma(\frac{n+2}{2})} = \frac{2(\sqrt{\pi})^n}{n \Gamma(\frac{n}{2})}$$

To use this formula you must know that $\Gamma(1) = 1$, $\Gamma(\frac{1}{2}) = \sqrt{\pi}$ and $\Gamma(x+1) = x \Gamma(x)$.

This formula is slightly unwieldy to use. It becomes more pleasant when we divide into odd and even n . Then the formulas become after easy simplification:

$$\begin{aligned} V_{2n} &= \frac{\pi^n}{n!} \\ V_{2n+1} &= \frac{2^{2n+1} n! \pi^n}{(2n+1)!} \end{aligned}$$