

Problem Set Number 2

This homework reviews Laplace Transforms and systems of Differential Equations for the test.

1) Solve the differential equations using Laplace Transforms

a) $y'(t) + 4y(t) = \cos t + 4 \sin t$ and $y(0) = 3$ Ans: $y = \sin t + 3e^{-4t}$

b) $y''(t) + 4y'(t) + 5y(t) = 0$ and $y(0) = 3, y'(0) = -4$ Ans: $y = 3e^{-2t} \cos(t) + 2e^{-2t} \sin(t)$

c) $y''(t) + 4y'(t) + 5y(t) = 25$ and $y(0) = 9, y'(0) = -5$ Ans: $y = 5 + 4e^{-2t} \cos(2t) + 3e^{-2t} \sin(2t)$

d) $y''(t) + 3y'(t) + 2y(t) = -2e^{-2t}$ and $y(0) = 2, y'(0) = 1$
Ans: $y = 2te^{-2t} - e^{-2t} + 3e^{-t}$

e) $y''(x) + 4y = 24e^{2t}$ and $y(0) = 5, y'(0) = 6$
Ans: $y = 3e^{2t} + 2 \cos 2t$

f) $y''(x) + 4y = 4t$ and $y(0) = 0, y'(0) = 5$
Ans: $y = t + 2 \sin 2t$

2) Solve the system using Eigenvalues and Eigenvectors

$$\begin{aligned} \frac{dx}{dt} &= 5x + 6y & x(0) &= -7 \\ \frac{dy}{dt} &= -2x - 2y & y(0) &= 4 \end{aligned}$$

The answer is

$$\begin{aligned} x(t) &= -3e^t - 4e^{2t} \\ y(t) &= 2e^t + 2e^{2t} \end{aligned}$$

Sketch a graph of the solution curves.

3) Solve the system using Eigenvalues and Eigenvectors

$$\begin{aligned} \frac{dx}{dt} &= 2x - 2y & x(0) &= 1 \\ \frac{dy}{dt} &= x + 4y & y(0) &= 2 \end{aligned}$$

The answer is

$$\begin{aligned} x(t) &= e^{3t}(\cos t - 5 \sin t) \\ y(t) &= e^{3t}(2 \cos t + 3 \sin t) \end{aligned}$$

Sketch a graph of the solution curves.