

Problem Set Number 1

This homework reviews Laplace Transforms and systems of Differential Equations for the test.

1) Solve the differential equations using Laplace Transforms

a) $y'(x) - 3y(x) = 1$ and $y(0) = 2$ Ans: $y = -\frac{1}{3} + \frac{7}{3}e^{-3t}$

b) $y''(x) + 2y'(x) + 5y = 0$ and $y(0) = 3, y'(0) = -1$ Ans: $y = 3e^{-t} \cos(2t) + e^{-t} \sin(2t)$

c) $y''(x) + 2y'(x) + 5y = 15$ and $y(0) = 6, y'(0) = -1$ Ans: $y = 3 + 3e^{-t} \cos(2t) + e^{-t} \sin(2t)$

d) $y''(x) + 2y'(x) + 5y = 2 \cos t + 4 \sin t$ and $y(0) = 4, y'(0) = -5$
Ans: $y = \sin t + 3e^{-t} \cos(2t) + e^{-t} \sin(2t)$

e) $y''(x) + 4y = 5e^t$ and $y(0) = 3, y'(0) = -1$
Ans: $y = e^t + 2 \cos 2t - \sin 2t$

f) $y''(x) + 4y = 3 \sin t$ and $y(0) = 3, y'(0) = 5$
Ans: $y = \sin t + 3 \cos 2t + 2 \sin 2t$

2) Solve the system using Eigenvalues and Eigenvectors

$$\begin{aligned}\frac{dx}{dt} &= -4x + 2y & x(0) &= 7 \\ \frac{dy}{dt} &= -3x + y & y(0) &= 10\end{aligned}$$

The answer is

$$\begin{aligned}x(t) &= 3e^{-t} + e^{-2t} \\ y(t) &= 9e^{-t} + e^{-2t}\end{aligned}$$

Sketch a graph of the solution curves.

3) Solve the system using Eigenvalues and Eigenvectors

$$\begin{aligned}\frac{dx}{dt} &= -x - 4y & x(0) &= 7 \\ \frac{dy}{dt} &= x - y & y(0) &= 10\end{aligned}$$

The answer is

$$\begin{aligned}x(t) &= -20e^{-t} \sin 2t + \frac{14}{2}e^{-t} \cos 2t \\ y(t) &= 10e^{-t} \cos 2t + \frac{7}{2}e^{-t} \sin 2t\end{aligned}$$

Sketch a graph of the solution curves.