

Pencil and Paper homework Number 9

This homework concerns Newton's method

1a) Find the three real roots of $x^3 - 4x^2 - 3x + 5 = 0$. Save in Calculator

b) Let r_1, r_2, r_3 be the three real roots. Find

1) $r_1 + r_2 + r_3$

2) $r_1r_2 + r_1r_3 + r_2r_3$

3) $r_1r_2r_3$

2) The line $y = x$ intersects the graph of $y = \tan x$ at infinitely many points. Find the first three positive intersections. For the second and third your initial estimate must be quite good because you are near an infinity of $\tan x$.

3a) The Babylonians knew in 1500 B. C. that $x_{n+1} = \frac{1}{2}(x_n + \frac{a}{x_n})$ would converge to \sqrt{a} . Derive this formula from Newton's method with $f(x) = x^2 - a$

b) Use the formula to find $\sqrt{17}$ (with initial value $x_1 = 4$) and $\sqrt{513}$ (with initial value $x_1 = 23$)

c)(optional) For those whose calculators can do complex numbers. Use the formula to find \sqrt{i} Use initial guess $x_1 = i$. When you have found x_5 square it and see if you get i . Can you recognize the decimals in x_5 ?

4) The first two positive intersections of e^x and $3.2 \sin x$ are close together. Find them. Do e^x and $3 \sin x$ have an intersection where x is positive?

5 (Illustrates trouble). The curves $y = x^3 - 6x^2 + x + 10$ and $y = -11x + 18$ intersect at $x = 2$. Pretend you don't know this and try to use Newton's method with an initial guess $x_1 = 2.1$. Note that it works, but v e r y s l o w l y.