

Pencil and Paper homework Number 7

This homework concerns implicit differentiation.

1) Using implicit differentiation find the derivatives of the following functions defined by relations.

a) $x^2 - y^2 = 1$

b) $x^3 - y^3 = 1$

c) $x^2 - 3xy + y^2 = 1$

d) $x + \sin xy = 1$

e) $y + e^{xy} = 1$

2) Find the equation of the tangent line to the curve at the given point. Make sure the point is on the curve. In one case it is not, and you don't have to do that problem, so it's worth identifying it.

a) $x^2 + y^2 = 13^2$ (5, 12)

b) $x^2 - xy + y^2 = 3$ (1, 2)

c) $x^3 - xy + y^2 = 3$ (1, 2)

d) $y^2 - x(x - 4)(x + 4) - 16 = 0$ (1, 1)

e) $x \sin y + y \sin x = \pi/2$ (1, $\pi/2$)

f) $xe^{xy} = e$ (1, 1)

3) Hydrogen is being blown into a cubical balloon at the rate of 4 cubic meters per minute. How fast is the surface area changing when the volume is 1728 cubic meters.

4) Batman is standing inside a conical tank 10 meters deep and 5 meters wide at the top. The Joker is running water into the tank at the rate of 1.5 cubic meters per minute. How fast is the water rising when it is up to Batman's head. We will take this height as 2 meters from the bottom (the pointy end) of the tank.

5) The volume of a donut is $2\pi^2 a^2 b$ where a is the radius of a cross section of donut and b is the distance from the center of the whole thing to halfway out in the part you eat. (Ask me to draw a picture in class.) a) How fast is the volume expanding if a is held fixed and b is expanding at 2 cm/min when $a = 8$ cm and $b = 17$ cm b) Now b is held fixed and a is expanding at 2 cm/min.

6) A street light is six feet high and 18 ft from a wall. A six foot high woman walks parallel to the wall at a rate of 1 ft/sec. She is 4 ft from the wall. How fast is her shadow moving along the wall when she is 5 ft past the point where she street light is closest to the wall.

7) Two runners are separated by 30 meters in an East-West direction at time $t = 0$. Allison runs north at 4 m/sec. Glen runs south at 3 m/sec. How fast are they separating at at time $t = 9$?

8) At time $t = 0$ a plane is 2 miles overhead. Charlie trains her telescope on the plane and tracks it. If the plane is going 180 miles/hr, how fast is the angle of the telescope from the vertical changing when $t = 1$ minute.

Answers(3-8, not in order): 1.28571, 10738, 2526, 1.909, 1.3333333, .46154, 6.3202