

Pencil and Paper homework Number 3

This homework concerns working out limits algebraically and with some limited additional tools.

1) Find the limits by factoring and cancelling. Not all exist.

$$a) \lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{x - 2} =$$

$$b) \lim_{x \rightarrow 1} \frac{x^2 - 5x + 4}{x - 1} =$$

$$c) \lim_{x \rightarrow 1} \frac{x^2 - 5x + 4}{x^2 - 3x + 2} =$$

$$d) \lim_{x \rightarrow 1} \frac{x^2 - 5x + 4}{x^2 - 2x + 1} =$$

$$e) \lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{x^2 - 5x + 4} =$$

$$f) \lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2} =$$

$$g) \lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 5x + 6} =$$

2) Put the given function into the limit and evaluate the limit.

$$a) f(x) = x^2 + 1 \quad \lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2} =$$

$$b) f(x) = x^2 - 2x + 1 \quad \lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1} =$$

3) Find the limits by multiplying by the conjugate.

$$a) \lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{x - 9} =$$

$$b) \lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{x - 3} =$$

$$c) \lim_{x \rightarrow 2} \frac{\sqrt{2x} - 2}{x - 2} =$$

$$d) \lim_{x \rightarrow -1} \frac{\sqrt{x^2 + 3} - 2}{x + 1} =$$

4) Put the given function into the limit and evaluate the limit.

$$a) f(x) = \sqrt{2x} \quad \lim_{x \rightarrow 8} \frac{f(x) - f(8)}{x - 8} =$$

$$b) f(x) = \sqrt{x+1} \quad \lim_{x \rightarrow 8} \frac{f(x) - f(8)}{x - 8} =$$

5) Find the limits by using the known limit $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

$$\begin{aligned} a) \lim_{x \rightarrow 0} \frac{\sin(3x)}{x} &= \\ b) \lim_{x \rightarrow 0} \frac{\sin(\frac{1}{2}x)}{x} &= \\ c) \lim_{x \rightarrow 0} \frac{\sin^2(x)}{x^2} &= \\ d) \lim_{x \rightarrow 0} \frac{1 - \cos(x)}{x^2} &= \end{aligned}$$

Hint: multiply numerator and denominator by $(1 + \cos x)$.

6) We have derived numerically that $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$. Use this fact to derive the following limits.

$$\begin{aligned} a) \lim_{x \rightarrow 0} \frac{e^{3x} - 1}{x} &= \\ b) \lim_{x \rightarrow 0} \frac{e^{\frac{1}{2}x} - 1}{x} &= \\ c) \lim_{x \rightarrow 1} \frac{e^x - e}{x - 1} &= \\ d) \lim_{x \rightarrow a} \frac{e^x - e^a}{x - a} &= \quad a \text{ any constant} \end{aligned}$$

Hint: For c) set $y = x - 1$ and rewrite the limit in terms of y .
This will work fine for d) also.