Pencil and Paper homework Number 14

This homework concerns Definite Integrals and Area one more time. Do all the problems using the fundamental theorem of Calculus. Many of these are basically substitution problems, and to get credit you must show the substitution and how you used it. Don't just have your calculator work out the answer. Make it look like you did it by hand.

1) Find the area under the curve $f(x) = 1/\sqrt{1-x^2}$ from 0 to 1/2

2) Find the area under the curve $f(x) = x^3/\sqrt{1-x^4}$ from 0 to 1/2

3) Find the area under the curve $f(x) = x/\sqrt{1-x^4}$ from 0 to 1/2

4) Find the area between the $x-$axis and the curve $f(x) = -x^2 + 4$

5) Find the area above the $x-$axis and below the curve $f(x) = x^3 - 4x$

6) Find the area above the $x-$axis and below the curve $f(x) = 8x - x^4$

7) Find the area above the $x-$axis and below the curve $f(x) = x^2 \sin(x^3)$ between 0 and the first time the curve comes back down and hits the x axis. This one is a little harder.

8) Now just some random practise.

a) $\int_0^{2\pi} \sin(\frac{1}{2}\theta)\, d\theta$

b) $\int_0^2 e^{2x} \sin(e^{2x})\, dx$

c) $\int_0^2 \frac{1}{4+x^2}\, dx$

da) $\int_0^1 \frac{1}{1+x^2}\, dx$

db) $\int_0^{10} \frac{1}{1+x^2}\, dx$

dc) $\int_0^{100} \frac{1}{1+x^2}\, dx$ It's worth noticing that it's a lot quicker to do the integral and then compute the answer than to do the integral on the caculator as a definite integral.

dd) What is $\pi/2$ as a decimal? How does it compare to your previous answer.

e) $\int_0^1 \frac{x+1}{x^2+2x+1}\, dx$

f) $\int_0^1 \frac{x+1}{\cos(x^2+2x+1)}\, dx$ Remember $1/\cos(\theta) = \sec(\theta)$

9) And more; remember

$$\sin^2\theta = \frac{1-\cos 2\theta}{2} \qquad \cos^2\theta = \frac{1+\cos 2\theta}{2} \qquad \sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$$

a) $\int_0^{\pi/2} \sin^2 x \cos x\, dx$

b) $\int_0^{\pi/2} \sin^2 x \cos^2 x\, dx$

c) $\int_0^{\pi/2} \sin^3 x \cos x\, dx$

d) $\int_0^{\pi/2} \sin^3 x \cos^2 x\, dx$