Pencil and Paper homework Number 12

This homework concerns Definite Integrals and Area.
For this homework all answers must have

at least 5 DECIMAL PLACES. Also you are to draw a graph for each problem indicating the points of division.

It is useful to remember that you can accumulate one thing after another in the calculator by (new thing) $+ x \to x$, which adds the new thing to whatever is already in x. This means you don't have to do too many calculations at once.

1) Find approximations to the Area under the Curve using the given function and the number n of subdivisions and the given a and b. Find LHS(n), RHS(n), TRAP(n) = (LHS(n)+RHS(n))/2, MID(n), and SIMP(n) = (2·MID(n)+TRAP(n))/3

a) $f(x) = x^2$ and $a = 0$, $b = 3$, $n = 3$

b) $f(x) = x^2$ and $a = 0$, $b = 3$, $n = 6$

Notice the improvements in the accuracy when the number n of subdivisions increases. Assume that the answer you got from SIMP is perfectly accurate, and call this number A.

c) For a) above find the errors LHS$-A$, TRAP$-A$, MID$-A$.

d) Same as c) but for b) above.

e) What effect did doubling the number n of intervals have on the errors?

2) Same as 1) but now the function is $f(x) = \sin(x)$, $a = 0$ and $b = \pi/2$, and for a) $n = 4$ and for b)$n = 8$. Then do c), d, e). For c),d),e) you can round off the value of SIMP to make it simple.

3) Same as 1) but now the function is $f(x) = e^x$, $a = 0$ and $b = 1$, and for a) $n = 4$ and for b)$n = 8$. You need not do c),d),e) for this problem.

4) Now the function is $f(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$, $a = 0$ and $b = 1$. Just find LHS, RHS TRAP MID and SIMP for $n = 4$. Be careful that you calculate $\frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$ correctly. For example, for $x = 3/8$ we have

$$\frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(3/8)^2} = \frac{1}{\sqrt{2\pi}} e^{-9/128} = .371855$$

Now multiply 2 times SIMP to get .68269 . You have just calculated the number of scores within one standard deviation of the mean for the bell curve distribution. Moreover, there is no other way to get this number.

5) Now the function is $f(x) = \frac{1}{x^2+1}$. Find the area under the curve from $x = 0$ to $x = 1$. Use $n = 4$. If your calculator enjoys it you might try to stay with fractions but it's easier to use decimals. When you get done, multiply the area by 4. What did you just find?