Pencil and Paper homework Number 11c

This homework concerns Newton's method for solving equations.
Let's say we want to solve $f(x) = a$. We form the new function $g(x) = f(x) - a$. Then you get the recursion formula from $g(x)$ by

$$x_{n+1} = x_n - \frac{g(x)}{g'(x)}$$

You have to guess x_0 to get started with. This is usually done with a graph. However, since this is stressful I guess it for you.
1) Solve the following equations:

a) $f(x) = x^2$ solve $f(x) = 2$ $x_0 = 1$

b) $f(x) = x^3$ solve $f(x) = 2$ $x_0 = 1$

c) $f(x) = x^3 + x$ solve $f(x) = 3$ $x_0 = 1$

d) $f(x) = \sin x - \frac{1}{2}x$ solve $f(x) = 0$ $x_0 = 2$

e) $f(x) = e^x + x$ solve $f(x) = 3$ $x_0 = 1$

Graph the three functions $\tan(x)$, x, $\tan x - x$ for $0 \le x \le 3\pi$

f) $f(x) = \tan x - 2x$ solve $f(x) = 0$ $x_0 = 1.5$

g) $f(x) = \tan x - 2x$ solve $f(x) = 0$ $x_0 = 4.6066$

h) $f(x) = \tan x - 2x$ solve $f(x) = 0$ $x_0 = 7.76032$

2) Show that the recursion formula for $f(x) = x^2$, solve $f(x) = a$ gives that x_{n+1} is the average of x_n and a/x_n. This was the way the Babylonians did square roots. The result you are trying to get is

$$x_{n+1} = \frac{1}{2}\Big(x_n + \frac{a}{x_n}\Big)$$

a) Use this method to find $\sqrt{19}$. Start with $x_0 = 4$

b) Use this method to find $\sqrt{1111}$. Start with $x_0 = 30$