Pencil and Paper homework Number 10

This homework concerns Curvature
The formula for curvature is

$$\kappa(x) = \frac{\frac{d^2y}{dx^2}}{[1 + (\frac{dy}{dx})^2]^{\frac{3}{2}}} = \frac{f''(x)}{[1 + f'(x)^2]^{\frac{3}{2}}}$$

When it says to graph $\kappa(x)$, use your calculator!

1) Find the curvature $\kappa(x)$ for $f(x) = x^2$. Graph $f(x)$ and $\kappa(x)$ from -4 to 4. Where is the curvature greatest and what is the value of the curvature when it is greatest?

2) Find the curvature $\kappa(x)$ for $f(x) = \sin(x)$. Graph $f(x)$ and $\kappa(x)$ from $-\pi$ to π. Looking at the graph, where is the curvature of $\sin(x)$ greatest? Where is it least?

3) Find the curvature $\kappa(x)$ for $f(x) = e^x$. Graph $f(x)$ and $\kappa(x)$ from -2 to 2. Looking at the graph, where is the curvature of e^x greatest? Is there a place where it is least?

4) Find the curvature $\kappa(x)$ for $f(x) = \cosh(x)$. This time you are required to *simplify* your result; the denominator can be simplified by using a hyperbolic identity. Do this. Graph $f(x)$ and $\kappa(x)$ from -2 to 2. Looking at the graph, where is the curvature of $\cosh(x)$ greatest?

5) Find the curvature $\kappa(x)$ for the line $f(x) = ax + b$. Is this a surprise?

6) What is the curvature $\kappa(x)$ for any curve $f(x)$ at a point of inflection of the curve? Explain.

The parametric formula for curvature is

$$\kappa(t) = \frac{\frac{dx}{dt}\frac{d^2y}{dt^2} - \frac{dy}{dt}\frac{d^2x}{dt^2}}{[(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2]^{\frac{3}{2}}} = \frac{x'(t)y''(t) - x''(t)y'(t)}{[x'(t)^2 + y'(t)^2]^{\frac{3}{2}}} = \frac{\begin{vmatrix} x'(t) & y'(t) \\ x''(t) & y''(t) \end{vmatrix}}{[x'(t)^2 + y'(t)^2]^{\frac{3}{2}}}$$

7) What is the curvature $\kappa(t)$ for the curve $x(t) = t^2,\ y(t) = t^3$?

8) What is the curvature $\kappa(t)$ for the circle

$$\begin{aligned} x &= a\cos(\omega\, t) \\ y &= a\sin(\omega\, t) \end{aligned}$$

You know this comes out $1/a$.

9) And we can do an ellipse with almost no extra effort, although it doesn't simplify so nicely

$$\begin{aligned} x &= 5\cos(t) \\ y &= 4\sin(t) \end{aligned}$$

Give a value of t where the curvature is maximum and a value of t where the curvature is minimum. You can do this just by eyeballing the formula for curvature. Where is the denominator smallest and largest?

And we should have one theorem of the mean problem.

10) $a = 0,\ b = 9,\ f(x) = \sqrt{x}$ find c and draw graph illustrating the situation.