

Pencil and Paper homework Number 1

1 a) Graph the function $f(x) = \ln|x^2 - 4|$. This is trickier than it seems. Ask yourself what happens as $x \rightarrow 2$. Your calculator will not show it accurately unless you zoom in on 2. b) Use your calculator skills to find the x for which $f(x) = 1$. This would be an excellent time to learn to use the solver on your calculator but you *can* do this by zooming in, which is a cruder technique. Hint: there are four values of x . c) Now solve the equation $f(x) = 1$ by algebraic techniques. This is only slightly tricky.

2 a) Graph the function $f(x) = \frac{x}{\sqrt{x^2-4}}$. Functions rarely just stop in midair so you might want to look closely at what happens if this seems to happen. Draw in any vertical asymptotes b) What are the domain and range of $f(x)$. Don't depend too much on your calculator; think about what happens. Can $f(x) = 1$ c) For what x is $f(x) = 2$? d) Can you solve this by algebraic techniques?

3 a) Graph the function $f(x) = x - \ln(x)$. Think about what happens to $\ln(x)$ as $x \rightarrow 0$. Is the calculator showing this to you adequately. Help the calculator by drawing what the graph really is. b) find the domain and range of $f(x)$. To do this you need to use the Trace button to find out how low it gets, and then compute $f(x)$ for the value of x you guess from the calculator. What is the (x, y) for the low point on the graph. Now you know the domain and range. c) Use your calculator to find the two points where $f(x) = 2$. You can use trace (which is crude) or the solver, but another useful trick is to graph both $f(x)$ and $g(x) = 2$ and find the intersection of the two curves. See your computer's manual or ask another student how to do this, which is a bit different on different calculators. This intersection skill is very handy. d) Now think about whether you could solve this problem algebraically. Do you have any chance at all?

4 a) Find all solutions to $\sin(3x) = .221$. Be sure to include the $2n\pi$ stuff when processing. Answer should be expressed as a decimal plus stuff with n and π and some fractions. b) Find all the solutions of the equation which lie between 0 and 2π and express these in decimal form. See if you can guess in advance how many solutions you will get. (Because you see 2π you know that this should be all in RADIANS, Check your answers on the calculator.)

5 a) Using your calculator find all solutions of $\sin(x) = .5x$. How many solutions are there? b) Using your calculator find all solutions of $\sin(x) = .08x$. Did the game change. Why?. c) Find all solutions of $\sin(x) = 2x$

6 a) Find the inverse function of $f(x) = e^{x^3-1} = \exp(x^3 - 1)$. This is straightforward. Less straightforward is b) find the inverse function of $f(x) = e^{x^2-1} = \exp(x^2 - 1)$. Do you have to trim this function first before you can find an inverse function? What are the domain and range of the trimmed function?

7 a) Draw a graph of the inverse function of $f(x) = x + \sin(x)$. b) Explain why $g(x) = x + 1.2\sin(x)$ has no inverse function. Hint: Use TRACE and go back and forth in the neighborhood of the flat part. Is the function one to one?

8 Here's an actual problem from Quantum mechanics. Think about the solution of $\tan(x) = 2x$. a) Draw the graph. b) Find the first three positive solutions. c) Have you got any chance of coming up with a formula for the solutions; yes or no? d) As the positive solutions increase, do they approach any friendly numbers?