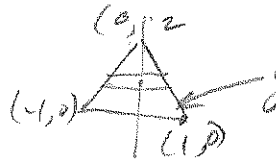
Fake Test

1. Length of $\cosh x$ from 0 to 1
2. Rotate e^{2x} around x axis from 0 to ∞ — Volume
3. Work — spring has $k = 2$ $\frac{\text{newtons}}{\text{meter}}$ $m = 4$
   Find work stretching from 0 to 3 — $\int_0^3 \text{Force}\, dx$
4. Find Center of mass of triangle shown



$y = -2x + 2$

$\frac{y-2}{-2} = x$

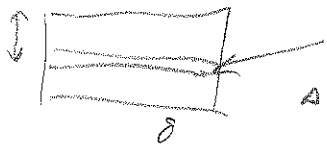
$\rho = 1$ Area $= \frac{1}{2} \cdot 2 \cdot 2 = 2$

$$\int y\, dA = \int_0^2 y(2x)\, dy = \int_0^2 y \frac{y-2}{-2} \cdot 2\, dy$$

$$= -\int_0^2 y^2 - 2y = -\left[\frac{1}{3}y^3 - y^2\right]_0^2 = -\left[\frac{8}{3} - 4\right] \quad (4 = \tfrac{12}{3})$$

$$= \frac{4}{3} \qquad \bar{y} = \frac{\frac{4}{3}}{2} = \frac{2}{3}$$

5. Force on Aquarium wall 3 meters high and 8 meters long



$\Delta A = h \Delta h$

$\Delta m = \rho \Delta V = \rho 8 \Delta A$

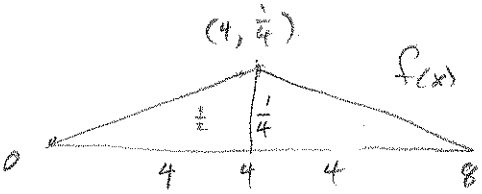
$\Delta F = g \rho \Delta A = g \rho 8 h \Delta h$

$W = g\rho 8 \int_0^3 h\, dh$

$= 8g\rho \frac{1}{2} h^2 \Big|_0^3$

$= 8g\rho \frac{9}{2}$

6.



$$f(x) = \begin{cases} \frac{1}{16}x & 0 \le x \le 4 \\ \frac{1}{2} - \frac{1}{16}x \end{cases}$$

$P(2 \le x \le 6)$

$$= \int_2^6 f(x)\, dx = 2\int_2^4 f(x)\, dx$$

$$= 2\int_2^4 \frac{1}{16}x\, dx = \frac{1}{8}\left[\frac{1}{2}x^2\right]_2^4$$

$$= \frac{1}{16}(16 - 4) = \frac{12}{16} = \frac{3}{4}$$

$$\text{mean} = \int_0^8 x f(x)\, dx$$

$$= \int_0^4 x \frac{1}{16}x\, dx + \int_4^6 x\left(\frac{1}{2} - \frac{1}{16}x\right) dx$$

$$= \frac{1}{16}\frac{1}{3}x^3\Big|_0^4 + \left[\frac{1}{2}\frac{1}{2}x^2 - \frac{1}{16}\frac{1}{3}x^3\right]_4^8$$

$$= \frac{64}{48} + \frac{1}{4}(64 - 16) - \frac{1}{48}(8^3 - 4^3) = 4$$

FT 2

7. a) $\frac{d^2y}{dt^2} - ky = 0$ b) $\left(\frac{dy}{dt}\right)^2 + y = 0$ c) $\frac{dy}{dt} + 2xy = x^2$

d) $\frac{dy}{dt} + ky^2 = 0$ e) $y\frac{dy}{dt} + y^2 = x$

a) which are linear
b) which are first order
c) which are 2nd order
d) which are homogeneous
e) Are any Quasilinear?

8. Draw slope field for $\frac{dy}{dx} = \frac{1}{2}y$ and sketch solution for $y(0) = 1$

9. Solve $\frac{dy}{dt} = \frac{1}{2}y$, $y(0) = 1$ by separation of vars

10. Use Euler's method to approximate $y(.5)$ w/ $\Delta t = .1$ for $\frac{dy}{dt} = \frac{1}{2}y$ $y(0) = 1$

11. Solve for linear differential equation $y(0) = 2$

$\frac{dy}{dt} + 2ty = 2t$ $\qquad u = e^{\int 2t\,dt} = e^{t^2}$

$e^{t^2}\frac{dy}{dt} + 2te^{t^2}y = 2te^{t^2}$

$\frac{d}{dt}(e^{t^2}y) = 2te^{t^2}$

$e^{t^2}y = \int 2te^{t^2}dt + c = e^{t^2} + c$

$y = 1 + ce^{-t^2}$