Pencil and Paper homework Number 16

This homework concerns the applications of vectors to three dimensional geometry.

1) Find the equation of the plane perpendicular to the vector $\vec{n}$ and through the point given.

a) $\vec{n} =< 1, 1, 1 >$ and point $P(1, 1, 1)$

b) $\vec{n} =< 2, 1, 3 >$ and point $P(1, 0, 2)$

c) $\vec{n} =< 2, 1, 0 >$ and point $P(3, -1, -2)$

d) $\vec{n} =< 0, 1, 0 >$ and point $P(3, 3, 3)$

2) Find the equation of the plane through the three points given

a) $P(1, 0, 0)$, $Q(0, 1, 0)$, $R(0, 0, 1)$

b) $P(1, 1, -2)$, $Q(2, -2, 5)$, $R(3, 0, -3)$

c) $P(4, -1, -3)$, $Q(5, -3, 2)$, $R(2, -2, 1)$

d) $P(4, -1, -3)$, $Q(5, -3, 2)$, $R(7, -7, 12)$ Surprise!

3) Find a point P on the given plane and a normal vector $\vec{n}$ to the plane.

a) $x - 3y - 2z = 7$

b) $4x + y - 3z = 31$

c) $3x - z = 11$

4) Find the distance from the point to the plane given by the equation. One way to do this is to pick any point Q in the plane, find $\vec{a} = \vec{QP}$ and then find $\text{comp}_{\vec{n}}\vec{a}$. Ignore sign.

a) $P(1, 2, 3)$, $\quad 2x - 3y + z = 4$

b) $P(-3, -2, 2)$, $\quad x + 3y + z = 2$

c) $P(1, 2, -5)$, $\quad x + 3y + z = 2$ ha!

d) $P(0, 0, 0)$, $\quad x + y + z = 1$

5) Find the equation of the line in all three forms.

a) Line through $P(1, 2, 3)$ and $Q(4, 1, 1)$

b) Line which is through $P(3, -2, 3)$ and has vector $\vec{v} = 2\hat{i} - 5\hat{j} + \hat{k}$

c) Line which is the intersection of the planes $x + 3y + z = 2$ and $3x + 2y - 3z = 5$

6) Find the distance from the point $P(2, 1, -3)$ to the line $< 1 + 3t, 4 + t, 3 - 2t >$. You first find a vector $\vec{m}$ perpendicular to the plane containing the point and the line. You next find a vector $\vec{p}$ perpendicular to both $\vec{m}$ and the line. Finally, you take a vector from P to any point on the line and find its component on $\vec{p}$. That is the distance (ignore sign).

7) Find

$$\vec{v}, \quad \frac{ds}{dt}, \quad \hat{T}, \quad \hat{N}, \quad \vec{a}, \quad a_T, \quad a_N, \quad \kappa$$

for

a) $\vec{r}(t) =< t, t^2 >$

b) $\vec{r}(t) =< \cos^2 t, \sin^2 t >$

Answers:
For a)

$$\hat{N} = \frac{1}{\sqrt{1+4t^2}} < -2t, 1 > \qquad a_N = \frac{2}{\sqrt{1+4t^2}} \qquad \kappa = \frac{2}{(1+4t^2)^{\frac{3}{2}}}$$

For b) (remember $2\sin t \cos t = \sin 2t$)

$$\hat{N} = \frac{1}{\sqrt{2}} < -1, -1 > \qquad a_T = 2\sqrt{2}\cos 2t \qquad a_N = 0 \qquad \kappa = 0$$

For kicks, graph b) on your calculator. Ha ha. This was predictable from $\kappa = 0$