Pencil and Paper homework Number 13A

This problem set covers some simple Taylor's series.

Start from $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$

1) Substitute $-x$ for x and get a series for e^{-x}.

2) Use $\sinh(x) = \frac{e^x - e^{-x}}{s}$ to get a series for $\sinh(x)$.

3) Find the series for $\cosh(x)$ by differentiating the series for $\sinh(x)$.

4) We will now use our knowledge of the series of $\cosh(x)$ and $\sinh(x)$ to get the series for $\cos(x)$ and $\sin(x)$.

a) Use $\cos(x) = \frac{e^{ix} + e^{-ix}}{2} = \cosh(ix)$ to get the series for $\cos(x)$ from the series for $\cosh(x)$. That is, put ix for x in the series for $\cosh(x)$. Remember $i^0 = 1$, $i^1 = i$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$, $i^5 = i$ and then it cycles through again.

b) Find the series for $\sin(x)$ by differentiating the series for $\cos(x)$. This will give you $-\sin(x)$; correct the sign.

5) We are going to use a series to show that $\sin(kx)$ is a solution of differential equation $\frac{d^2y}{dx^2} + k^2 y = 0$.

a) Find the series for $\sin(kx)$ by substituting kx for x. Call this series $y(x)$.

b) Find $\frac{d^2y}{dx^2}$. This is, Take two derivatives of the series you found in a). Simplify as you go.

c) Substitute the series from b) and the series from a) into the differential equation $\frac{d^2y}{dx^2} + k^2 y = 0$. You should get 0.