

Pencil and Paper homework Number 13

This problem set covers the Alternating series test and Absolute Convergence.

1) Use the Alternating Series Test to check for convergence in the following series. Not all these series are alternating, so say *TEST DOES NOT APPLY* when you come across these. Also indicate whether the series is absolutely or conditionally convergent.

a) $\sum_1^{\infty} \frac{(-1)^{k+1}}{k}$

b) $\sum_1^{\infty} \frac{(-1)^{k+1}}{k^2}$

c) $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{5}} - \frac{1}{\sqrt{6}} + \dots$

d) $1 + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{5}} + \frac{1}{\sqrt{6}} - \dots$

e) $\sum_1^{\infty} \frac{\cos k\pi}{k}$

f) $\sum_1^{\infty} \frac{\cos k\pi}{k^2}$

g) $\frac{3}{1!} - \frac{3^2}{2!} + \frac{3^3}{3!} - \frac{3^4}{4!} + \frac{3^5}{5!} - \frac{3^6}{6!} + \dots$

2) Error Estimates

a) The series $\sum \frac{(-1)^{k+1}}{k^{\frac{2}{3}}}$ converges conditionally. Estimate the error when you use the first 50 terms.

b) Same series $\sum \frac{(-1)^{k+1}}{k^{\frac{2}{3}}}$ but the first 500 terms.

b) Same series $\sum \frac{(-1)^{k+1}}{k^{\frac{2}{3}}}$ but now figure out how many terms you need so that the error will be less than .001
ans: 31,622 terms are enough. The sum is .635678 for that many terms.

3) More accurate Error Estimates. The series

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{2^k}$$

is both a geometric series and an alternating series.

a) Find the sum of the series from $k = 0$ to $k = 10$

b) Use the alternating series estimate to estimate the size of the error.

c) Use the formula for summing a geometric series to find the EXACT error. This of course is

$$\sum_{k=11}^{\infty} \frac{(-1)^k}{2^k}$$