

Pencil and Paper homework Number 11

This problem set covers the Integral Test.

1) Use the Integral Test to check for convergence in the following series. If they converge, use the following to estimate the size of the sum

$$a_1 + \int_2^{\infty} f(x) dx \leq \sum_{i=1}^{\infty} a_i \leq a_1 + \int_1^{\infty} f(x) dx$$

a) $\sum_1^{\infty} \frac{1}{(i+1)\ln(i+1)}$

b) $\sum_1^{\infty} \frac{i}{1+i^2}$

c) $\sum_1^{\infty} \frac{1}{1+i^2}$

d) $\sum_1^{\infty} \frac{1}{e^i}$

e) $\sum_1^{\infty} \frac{i^3}{e^i}$

2) Use the formula

$$a_1 + \int_2^{l+1} f(x) dx \leq \sum_{i=1}^l a_k \leq a_1 + \int_1^l f(x) dx$$

to estimate (trillion = 10^{12} , million = 10^6)

a) $\sum_1^{\text{trillion}} \frac{1}{i}$

b) $\sum_1^{\text{trillion}} \frac{1}{(i+1)\ln(i+1)}$

c) $\sum_1^{\text{million}} \frac{1}{\sqrt{i}}$

3) Use the p-series test to test the convergence of (notice the first three have exponents which are decimal numbers 2.2, 1.2, .8)

a) $\sum_1^{\infty} \frac{1}{i^{2.2}}$

b) $\sum_1^{\infty} \frac{1}{i^{1.2}}$

c) $\sum_1^{\infty} \frac{1}{i^{.8}}$

d) $\sum_1^{\infty} \frac{1}{\sqrt{i}}$

e) $\sum_1^{\infty} \frac{1}{i\sqrt[3]{i}}$

4) Use the comparison test (comparing with p-series) to test for convergence

a) $\sum_1^{\infty} \frac{1}{1+i^2}$

b) $\sum_1^{\infty} \frac{1}{i^{-\frac{1}{2}}}$

c) $\sum_1^{\infty} \frac{1}{\sqrt{i}-1}$