

Pencil and Paper homework Number 10

This problem set covers the some infinite series.

PARTIAL SUMS

1) Find the first 8 partial sums for each of the following series. Give the limit of the partial sums if you think you know it.

a) $\sum_1^{\infty} \frac{1}{2^j}$

b) $\sum_1^{\infty} \frac{1}{5^j}$

c) $\sum_1^{\infty} \frac{1}{j}$

d) $\sum_1^{\infty} \frac{1}{j^2}$

e) $\sum_1^{\infty} \left(\frac{1}{j} - \frac{1}{j+1}\right)$

LIMITS OF PARTIAL SUMS

2) Find a formula for the partial sums for each of the following series and then take the limit of s_n to find the sum of the series.

a) $\sum_1^{\infty} \frac{1}{2^j}$

b) $\sum_1^{\infty} \frac{1}{5^j}$

c) $\sum_1^{\infty} \left(\frac{1}{j} - \frac{1}{j+1}\right)$

d) $\sum_1^{\infty} \left(\frac{1}{j(j+1)}\right)$ (Find a few partial sums)

GEOMETRIC SERIES

3) The most important series is the GEOMETRIC SERIES, which has a formula for the sum which works when $|r| < 1$. The formula is

$$\sum_{j=0}^{\infty} r^j = 1 + \frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \dots = \frac{1}{1-r}$$

In the following identify which ones are geometric series (and identify r), which ones which are geometric series have $|r| < 1$, and for these find the sum. Notice that nobody promises that the geometric series will start with 1; you may have to compensate if it starts somewhere later (or earlier) than it should.

a) $1 - 6 + 36 - 216 + 1296 - \dots$

b) $1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \dots$

c) $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} + \dots$

d) $9 + 3 + 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \dots$

e) $\frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \frac{1}{1024} + \dots$

f) $1 + \frac{1}{2} + \frac{1}{8} + \frac{1}{64} + \frac{1}{1024} + \dots$

4) In the following problems a) mindlessly use the Geometric Series formula to get the sum of each series and then b) find for which x the condition $|r| < 1$ is valid.

a) $1 + 2x + 4x^2 + 8x^3 + 16x^4 + \dots$

b) $1 - 2x + 4x^2 - 8x^3 + 16x^4 + \dots$

c) $1 + x^2 + x^4 + x^6 + x^8 + \dots$

d) $1 - x^2 + x^4 - x^6 + x^8 + \dots$

e) $1 + 2x^2 + 4x^4 + 8x^6 + 16x^8 + \dots$

f) $1 - 2x^2 + 4x^4 - 8x^6 + 16x^8 + \dots$

g) $1 + \sqrt{2}x + 2x^2 + 2\sqrt{2}x^3 + 4x^4 + \dots$

h) $1 - \sqrt{2}x + 2x^2 - 2\sqrt{2}x^3 + 4x^4 + \dots$

i) $1 + \sin^2 x + \sin^4 x + \sin^6 x + \sin^8 x + \dots$ Ans: $\sec^2 x$

j) $1 + \sin x + \sin^2 x + \sin^3 x + \sin^4 x + \dots$ Ans: $\sec x(\sec x + \tan x)$

k) $1 - \tan^2 x + \tan^4 x - \tan^6 x + \tan^8 x + \dots$ Ans: $\cos^2 x$ for $x < ?$.