

Pencil and Paper homework Number 7

This problem set has some applications of calculus to probability and statistics.

1) Let the probability density function be

$$f(x) = \begin{cases} cx(4-x) & \text{for } 0 \leq x \leq 4 \\ 0 & \text{elsewhere} \end{cases}$$

- Determine c so that $\int_{-\infty}^{\infty} f(x) dx = 1$
 - Sketch the graph of the probability density function.
 - Find $P(1 \leq x \leq 3)$, the probability that x is between 1 and 3.
 - Find $P(2 \leq x \leq 3)$, the probability that x is between 2 and 3.
 - Find $P(2 \leq x \leq 4)$, the probability that x is between 2 and 4.
 - Use the calculus formula to find the most probable value of x (that is, the mean μ ,) which is obviously 2.
- 2) Let the probability density function be

$$f(x) = \begin{cases} \frac{1}{12}x & \text{for } 0 \leq x \leq 4 \\ -\frac{1}{6}x + 1 & \text{for } 4 \leq x \leq 6 \\ 0 & \text{elsewhere} \end{cases}$$

In the following problems it is permitted to use brains rather than knee-jerk calculation.

- Verify that $f(x)$ is a probability density function.
 - Find $P(1 \leq x \leq 4)$
 - Find $P(1 \leq x \leq 3)$
 - Find $P(4 \leq x \leq 6)$
 - Find $P(4 \leq x \leq 5)$
 - Find $P(1 \leq x \leq 5)$
 - Find $P(3 \leq x \leq 5)$ Ans: $\frac{13}{24}$.
 - Find the mean of the distribution. Ans: $\frac{10}{3}$.
- 3) Next you are going to find the probability that a score is within 1.5σ of the mean. Since it doesn't matter where the mean is or what the standard deviation is, we may as well use the standard bell curve probability density function (invented by Carl Johann Friederich Gauss for stellar observations). You have to use your calculator to do these integrals; there is no antiderivative.

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

- a) Verify that $f(x)$ is a probability density function. In this case you may use 5 as an approximation to ∞ .
- b) For this $f(x)$, we know $\sigma = 1$. Find $P(-\sigma \leq x \leq \sigma)$ The result is familiar.
- c) Find $P(-2\sigma \leq x \leq 2\sigma)$. This is the probability that x is within two standard deviations of the mean.
- d) Find the probability that x is within 1.5 standard deviations from the mean.
- 4) The standard bell curve with mean 0 is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$$

- a) How high is $f(x)$ when $x = 0$?
- b) Graph $f(x)$ when $\sigma = \frac{1}{2}$, $\sigma = 1$ and $\sigma = 2$ on the same axis (use ZoomSqr so the axes are the same size) and draw it on your paper.