Pencil and Paper homework Number 6B

This problem set has some applications of integrals to geometry and physics.

1) Here are some problems where you find the area of a surface of revolution. The formula is $dS = 2\pi y ds$ when you are going around the x axis. Find the area of the surface of revolution generated by revolving the given curve around the x axis between the given limits. Use your calculator to do the integral when necessary.

a) $y = \frac{2}{3}x^{\frac{3}{2}}$ from $x = 0$ to $x = 2$

b) $y = e^x$ from $x = 0$ to $x = 2$

c) $x = a\cos t, \quad y = a\sin t \quad$ from $t = 0$ to $t = \pi/2$

2) Let's combine a couple of techniques

a) Find the volume of the solid found by rotating e^{-x} around the x axis from 0 to infinity. It converges.

b) Find the surface area of the object in a). After you set up the problem, let $u = e^{-x}$. Don't forget to change the limits, which will become finite and backwards. Reverse the limits using the extra - sign. You will now be at $2\pi\int_0^1 \sqrt{1+u^2}\,du$. You can do this with a formula, or, for fun, substitute $u = \tan\theta$ and get out old friend $\int_0^{\pi/4}\sec^3\theta\,d\theta$. You should then get $2.29559\pi = 7.2118$.

3) Same as 2) but this time the function is $y = 1/x$ from $x = 1$ to $x = \infty$. The volume integral converges.

a) Find the volume.

b) Show that the surface area is infinite by showing the integral is larger than an integral which diverges. Hint: $\sqrt{1+(\frac{dy}{dx})^2} \geq 1$

Notice this means you could pour a finite amount (π cubic meters) of paint into into it, then pour the paint back out and the infinite surface area would be painted.

4) Some centroid problems. Try to use symmetry to simplify some of the work.

a) Find the Centroid of an isoceles triangle with corners (-a,0),(a,0) and (0,h).

b) Find the Centroid of a quarter of a circle in the first quadrant: $x^2 + y^2 = a^2$ with $x \geq 0$ and $y \geq 0$.

c) Find the Centroid of a triangle with corners at (0,0), (a,0) and (a,b).

5) A mass problem. A quarter circle has mass density $\rho(x, y) = 2x$ and equation $x^2 + y^2 = a^2$ where $x \geq 0$ and $y \geq 0$.

a) Find the mass, using vertical stripes.

b) Find $\overline{x}$

c) Find $\overline{y}$

6) A tank of pig fat is formed from rotating the parabola $y = x^2$ around the y-axis and is 9 meters tall. Find the work needed to pump the pig fat out of the tank to the level of the top of the tank. Pig fat has density $\rho = 800\,\text{kg/m}^3$.

7) A swimming pool has glass sides and each side is a rectangle. All the angles are 90°. The long sides are 4 meters deep and 8 meters long. The short sides are 4 meters deep and 4 meters long. Find the force on one long side and one short side. Water has mass 1000kg/meter^3.

8) A trough for giant cows has a cross section in the shape of an isoceles triangle with height 4 meters and top 5 meters. The trough is filled completely. Find the force on the end of it.
b)Now the trough is filled with water to a height of 3.5 meters. Find the force on the end

9) A horse is attached to a spring and stretches it 10 cm. This horse is known to pull with a force of 50 Newtons. a) Find the spring constant k. b) Find the work done by the horse. c) Three more horses are attached and they pull until the spring is stretched 10 more cm. Find the work done by the four horses in stretching the spring from 10 cm to 20 cm. DO NOT FORGET to convert the cm to meters. 1 meter = 100 cm.

Some answers:

$\frac{4a}{3\pi}$ $\left(\frac{2a}{3}, \frac{b}{3}\right)$ 7.2118 $\frac{h}{3}$ 7.5 $\frac{2a^3}{3}$ $\frac{3\pi a}{16}$ $2,992,560$ $125,052$ $313,600$