Pencil and Paper homework Number 6A

This problem set has some applications of integrals to geometry and physics.

1) Do the following arc length problems by hand and then verify your answers with your calculator.

a) $y = \cosh x$ from $x = 0$ to $x = 2$

b) $y = \frac{2}{3}x^{\frac{3}{2}}$ from $x = 0$ to $x = 8$

c) $y = \frac{x^3}{3} + \frac{1}{4x}$ from $x = 1$ to $x = 2$

d) $x = \cos^3 t, \quad y = \sin^3 t \quad$ from $t = 0$ to $t = \pi/2$

2) Here are some uncooked funtions, so you will have to use your calculator to find the length.

a) $y = x^3$ from $x = 0$ to $x = 2$

b) $y = \sin x$ from $x = 0$ to $x = \frac{\pi}{2}$

c) $y = e^{-x}$ from $x = 0$ to $x = 2$

d) $y = \ln x$ from $x = 1$ to $x = 8$

3) Here are some problems where you find the area between two curves.

a) Between $y = x^2$ and $y = x^3$ from $x = 0$ to $x = 1$. I gave you the intersection points this time but for the rest of the problems you need to find them yourself.

b) Between $y = \cos(x)$ and $y = 1 - \frac{2}{\pi}x$.

c) Between $y = x^2$ and $y = x^4$.

d) Between $y = x$ and $y = x^3$. This is tricky because if you go from -1 to 1 you will get 0 because half of the area is positive and half negative. So do the positive half and double to get the right answer.

e) Between the circle of radius a and the line $y = x$.

3) Here are some problems where you find the volume of a surface of revolution.

a) Revolve $y = x^2$ around the y axis. Find the volume of the solid so formed from $y = 0$ to $y = 4$.

b) Revolve the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (really just its top half) around the x-axis and find the volume of the resulting ellipsoid. When you are done let $b = a$ and see if you get the volume of a sphere. This is actually a rather easy problem. If $a > b$ this is a prolate spheroid.

c) Revolve the line $y = \frac{a}{h}x$ around the x axis and find the volume of the object for $0 \leq x \leq h$. This is a cone and the answer should be one third the base times the height.

4) Here is a problem we do the same way but it is not a solid of revolution. We tip the great pyramid of Giza on its side so that it's nose is at the origin and the positive x axis goes through the center of the bottom. The bottom is a square of side $2a$ and the height (from 0 to the square along the x axis) is h. The parallel cross sections are all squares. The relevant line is $y = \frac{a}{h}x$. Find the volume, and once again it should be one third the base times the height.