

Pencil and Paper homework Number 5

This homework has some random integrals and some improper integrals.

1) Do the following routine integrals using tables. Most of these could be done by hand with a little patience.

a) $\int \frac{x dx}{3x+5}$

b) $\int_0^1 \frac{x dx}{3x+5}$

c) $\int \frac{dx}{x\sqrt{2x-3}}$

d) $\int_2^4 \frac{dx}{x\sqrt{2x-3}}$

e) $\int \frac{2x+3}{\sqrt{4x+5}} dx$

f) $\int \frac{x^3}{(x^2+9)^2} dx$

g) $\int \frac{x^2}{(x^2-9)^2} dx$

2) Here are some type I improper integrals.

a) $\int_0^\infty e^{-x} dx$

b) $\int_0^\infty \frac{1}{x^2+4} dx$

c) $\int_0^\infty \frac{x}{1+x^3} dx$

d) $\int_1^\infty \frac{1}{(x+2)^3} dx$

3) For statistics we need the function

$$\operatorname{erf}(x) = \frac{1}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

Use your calculator to find

a) $\operatorname{erf}(1)$

b) $\operatorname{erf}(\frac{1}{\sqrt{2}})$ Double this answer. Look familiar?

c) $\operatorname{erf}(2)$

d) $\operatorname{erf}(3)$

d) $\operatorname{erf}(5)$

What value do we appear to be approaching? What would you guess the value of

$$\operatorname{erf}(\infty) = \frac{1}{\sqrt{\pi}} \int_0^{\infty} e^{-t^2} dt$$

would be?

4) What is $\frac{d}{dx}\operatorname{erf}(x)$

5) In the following cases prove the integral converge or diverge by a comparison test. Indicate what the comparison integral is and why it converges.

a) $\int_1^{\infty} \frac{1}{x^4+1} dx$

b) $\int_0^{\infty} \frac{e^{-x}}{x^2+1} dx$

c) $\int_2^{\infty} \frac{x}{x^2-e^{-x}} dx$

d) $\int_0^{\infty} \frac{\sin x}{x^2+1} dx$

e) $\int_0^{\infty} e^{-x} \sin(x^2) dx$