Pencil and Paper homework Number 4B

This homework is really about numerical integration, but we use the opportunity to find the circumference of an Ellipse, which cannot be done by antiderivaties unless you know about elliptic functions, which almost nobody does.

The formulas for the arc length of a curve are

$$\begin{aligned} s &= \int \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx \qquad \text{rectangular form} \\ s &= \int \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dx \qquad \text{parametric form} \end{aligned}$$

The ellipse we will work with has the rectangular form equation

$$\frac{x^2}{2^2}+\frac{y^2}{1^2}=1$$

which has the parametric equations

$$\begin{aligned} x &= 2\cos t \qquad 0 \le t \le \frac{\pi}{2} \\ y &= 1\sin t \end{aligned}$$

1) Solve the rectangular form for y and take the derivative. Put into the arc length formula and simplify.

2) Use numerical integration with n=4 as if you were going to get the Simpson approximation where x goes from 0 to 2. This is the length of the upper right quarter of the ellipse. However, a suprise is in store. Usually we calculate LHS first but so you won't waste your time first Calculate RHS. When you put in $x = 2$ what terrible things happen?

Time to give up on the rectangular effort. Perhaps we will have better luck with the parametric effort.

3) Now use the parametric form to get the same length by integrating t from 0 to $\pi/2$. Again use $n = 4$ and the Simpson approximation. To help you check your work I mention that $\Delta x = \pi/8$ and RHS $= 2.61845$. The answer is 2.42812. OOPS, one of those digits is wrong. Find the right answer. This answer, using a mere $n = 4$ is off by one digit in the fifth decimal place.

Historical Note: This problem was very important in the history of Calculus because for the first time it proved impossible to find an antiderivative. It was bad day for Johann Bernoulli.