Pencil and Paper homework Number 3

This homework has a few more volume problems and some work problems.

1) Find the volume you get by rotating the upper half of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

around the $x-$axis. Ans: $\frac{4\pi ab^2}{3}$

2) Find the volume of the solid gotten by rotating the curve $y = \sqrt{x}$ around the $x-$axis from $x = 0$ to $x = 9$. Ans: $\frac{81\pi}{2}$

3) Find the volume of the solid gotten by rotating the curve $y = \sqrt[3]{x}$ around the $x-$axis from $x = 0$ to $x = 27$.

4) Find the volume of the solid gotten by rotating the curve $y = \sqrt[3]{x}$ around the $y-$axis from $x = 0$ to $x = 27$. Notice that this volume is ABOVE the curve. Ans: $\frac{2187\pi}{7}$

5) Use shells to do problem 4) and look closely at the picture to be sure you are using the right shells.

6) A square in the $xy-$ plane with center at $(8, 0)$ and sides of length 2 is rotated around the $y-$axis. Find the volume of the square donut

a) by using disks

b) by using shells

This problem is important when building Large Hadron Colliders.

In the following work problems don't forget the 9.8 m/sec^2 for gravity

7) Find the work done pushing a 5 kg mass up a plane inclined 30° to the horizontal for 3 meters along the plane. Ans: 73.5 Joule

8) A 5 kg weight stretches a spring 10 cm. (Spring hung from ceiling) This will determine the spring constant k Now compute how much work is done stretching the spring from equilibrium to 20 cm.

9) A 60 meter rope with density 2 kg/meter hangs off the edge of an 80 meter high doghouse (with flat roof). How much work is done pulling the rope up to the top of the doghouse? The dog's name is Fenrir. Ans: 3*,280 Joule (previous answer was wrong).

10) A tank with a 2 meter by 2 meter base and 6 meters high is sunk in the ground and filled with water weighing 1 gm/cm^3 How much work is done pumping out the water. Use horizontal slabs of water with thickness Δy, each one of which has to be raised to the top and they all go up different distances. Remember to convert the density to kg/m^3 first. Ans: 7**,600 Joule