

Pencil and Paper homework Number 2

This homework works with average values and surfaces of revolution.

1) Find the average value of the function over the interval.

a) $f(x) = x^2$ from 0 to 2 Ans: $\frac{4}{3}$

b) $f(x) = \sin x$ from 0 to π

c) $f(x) = \sin^2 4x$ from 0 to $\pi/4$

d) $f(x) = \sin^2 4x$ from 0 to $\pi/2$ Ans: $\frac{1}{2}$

e) $f(x) = 16 - 4x^2$ from -2 to 2

2) Find the area between the curves

a) $f(x) = 8 - x^2$ and $g(x) = x^2$

b) $f(x) = x^2$ and $g(x) = 3x - 2$ Ans: $\frac{1}{6}$

c) $f(x) = 2x^2$ and $g(x) = 4x$

d) $f(x) = \sin x$ and $g(x) = (\frac{2}{\pi})x$ Ans: $2 - \frac{\pi}{2}$ (Remember to do both sides.)

e) $f(x) = e^x$ and $g(x) = (e - 1) \cdot x + 1$

3) Find the area above the circle $x^2 + (y + 1)^2 = 2$ and below the circle $x^2 + y^2 = 1$. Do not try to do this integral by hand; do it on your calculator after setting it up. (This one is a little harder.)
Ans= 1

4) Find the volume of revolution described by using disks (=rings)

a) Rotating $y = \sqrt{x}$ around the x-axis from 0 to 4

b) Rotating $y = x^2$ around the y-axis from $x = 0$ to $x = 2$ Ans: 8π

c) Rotating $y = \sin x$ around the x-axis from 0 to π

d) Rotating $y = \cos x$ around the x-axis from $-\pi/2$ to $\pi/2$

e) Rotating $y = \cos x$ around the y-axis from $x = 0$ to $x = \pi/2$ Ans: $\pi^2 - 2\pi$

5) Find the volume of revolution described by using shells (=pipes)

a) Rotating $y = \cos x$ around the y-axis from $x = 0$ to $x = \pi/2$. (Same as 4e)

6) Rotate the curve $y = e^{-x}$ around the x axis from 0 to 1. what is the volume. Now go from 0 to 10, then from 0 to 100, then from 0 to 1000. Are you approaching some number? We will revisit this problem later when we get to improper integrals. Answer; the volumes approach $\pi/2$.