MAT 612 **HW #4** Name ____________________

03/05/10, due Friday 03/12/10

25 points

1. Prove: If P is a prime ideal, then P is an irreducible ideal.

2. Find a standard primary decomposition of the ideal $I = (x^2, xy, 2)$ in the ring $\mathbb{Z}[x]$.
Note: It might be handy to use the following characterizations: P is prime if and only if R/P is a domain; Q is primary if and only if every zero-divisor of R/Q is nilpotent.

3. Show that each of the following is a standard primary decomposition of the ideal $I = (x^2, xy)$ in $\Bbbk[x, y]$, $\Bbbk$ a field.

(i) $I = (x) \cap (x^2, y)$

(ii) $I = (x) \cap (x^2, x + y)$

(iii) $I = (x) \cap (x, y)^2$

4. Show that the following are primary decompositions in the ring $\mathbb{Z}[x]$, and determine whether they are standard.

(i) $(4, 2x, x^2) = (4, x) \cap (2, x^2)$

(ii) $(9, 3x + 3) = (3) \cap (9, x + 1)$