MAT 612 **HW #3** Name ________________________________

02/16/10, due Monday 02/22/10

25 points

1. Let R be a commutative ring with 1. Suppose $f\colon M \longrightarrow N$ is an R-module homomorphism, and K be an R-module. Show that f induces a homomorphism $f^*\colon \operatorname{Hom}_R(N,K) \longrightarrow \operatorname{Hom}_R(M,K)$. Show that $(\mathrm{id}_M)^* = \mathrm{id}_{\operatorname{Hom}_R(M,K)}$ and that $(f \circ g)^* = g^* \circ f^*$.

2. Suppose $0 \longrightarrow M \xrightarrow{f} N \xrightarrow{g} P \longrightarrow 0$ is an exact sequence of R-modules. Show that the induced sequence $0 \longrightarrow \operatorname{Hom}_R(P,K) \xrightarrow{g^*} \operatorname{Hom}_R(N,K) \xrightarrow{f^*} \operatorname{Hom}_R(M,K) \longrightarrow 0$ is exact at $\operatorname{Hom}_R(P,K)$ and at $\operatorname{Hom}_R(N,K)$.

3. Find an example to show that the induced sequence in Problem 2 need not be exact at $\mathrm{Hom}_R(M, K)$. *Hint: Let* $R = K = \mathbb{Z}$.

4. Exercise A.2.4 from Schenck.

5. Exercise A.2.5 from Schenck.