MAT 612 **HW #1** Name ____________________

01/29/10, due Wednesday 02/03/10

25 points

1. (a) Prove: if R is left-noetherian, then R^n is noetherian as a left R-module. (This is Exercise 12.1.15 - refer to that exercise for a hint.)

(b) Show that a finitely-generated module over a noetherian ring is noetherian.
Hint: First prove that a quotient of a noetherian module is noetherian.

2. Let R be a PID, with field of fractions F, and let S be a subring of F containing R. Prove that S is a PID.
Hint: First show, if $\frac{a}{b} \in S$ and a and b have no common prime factors, then $\frac{1}{b} \in S$.

3. (Exercise 12.1.6) Show: if R is an integral domain[1] and M is a non-principal ideal of R, then M is a torsion-free R-module of rank one, but is not a free R-module.

[1] in particular, R is commutative