MAT 511 **HW #7** Name ____________________

11/18/13, due Friday 11/22/13

25 points

1. Show any group of order 154 is solvable.

2. (a) Do the following special case of Problem 5.13 from Rotman: let P be a Sylow p-subgroup of G, and $H = \mathbf{N}_G(P)$. Then $\mathbf{N}_G(H) = H$.

   Hint: Show P is a characteristic subgroup of H.

   (b) Suppose G is a nilpotent group, and $H \leq G$. Suppose $\mathbf{N}_G(H) \neq G$. Prove $\mathbf{N}_G(H) \neq H$.

   Hint: Use the fact that $\mathbf{Z}(G) \neq 1$. There are two cases: $\mathbf{Z}(G) \subseteq H$ and $\mathbf{Z}(G) \not\subseteq H$.

   (c) Use parts (a) and (b) to show that every finite nilpotent group is isomorphic to the direct product of its Sylow subgroups.

3. (a) Do Problem 6.38 from Rotman.

(b) Use part (a) to show that a group G is nilpotent if and only if $G^n = 1$ for some $n \geq 1$. Recall $\{G^k \mid k \geq 1\}$ is the lower central series of G, defined by $G^1 = G$ and $G^{k+1} = [G, G^k]$ for $k \geq 1$.

4. Suppose R is a ring with 1, and $r \in R$. Prove the following statements are equivalent:

(i) $r \in I$ for every maximal left ideal I of R.

(ii) for every simple left R-module M, and every $x \in M$, $rx = 0_M$.

Hint: Compare with Exam 2, problem 5.