MAT 511 **HW #3** Name ____________________

9/16/13, due Friday 9/20/13

25 points

1. Let G be a group.

   (a) Prove that $\mathrm{Inn}(G)$ is a normal subgroup of $\mathrm{Aut}(G)$.

   (b) Prove that $\mathrm{Inn}(G)$ is isomorphic to $G/\mathbf{Z}(G)$, where $\mathbf{Z}(G)$ is the center of G.

   (c) Prove that $\mathrm{Inn}(G)$ cannot be cyclic, unless it is trivial. (Hint: Use (b).)

2. Give an example to show that the image of a homomorphism $\varphi\colon G \to H$ need not be a normal subgroup of H.

3. Let H and K be subgroups of G.

(a) Prove KH is a subgroup of G if and only if $KH = HK$.

(b) Prove KH is a subgroup of G if $H \subseteq \mathbf{N}_G(K)$.

4. Let G be a group and $H \leq G$. The *centralizer* of H is

$$\mathbf{C}_G(H) = \{x \in G \mid xh = hx \text{ for all } h \in H\}.$$

Prove that the quotient group $\mathbf{N}_G(H)/\mathbf{C}_G(H)$ is isomorphic to a subgroup of $\mathrm{Aut}(H)$.