MAT 511 **HW #2** Name ________________________

9/7/13, due Friday 9/13/13

25 points

1. Let G be a group and $g \in G$. Let $\rho_g \colon G \to G$ and $\lambda_g \colon G \to G$ be the functions defined by $\rho_g(x) = xg$ and $\lambda_g(x) = gx$ for $x \in G$. The functions $\rho \colon G \to S_G$, $g \mapsto \rho_g$ and $\lambda \colon G \to S_G$, $g \mapsto \lambda_g$ are injective homomorphisms. Let $R = \mathrm{im}(\rho) \subseteq S_G$ and $L = \mathrm{im}(\lambda) \subseteq S_G$ denote the images of ρ and λ, respectively. Each of these subgroups is isomorphic to G - that is Cayley's Theorem. Show that R is equal to the centralizer of L in the group S_G.[1]

2. Do Problem 1.36 from the text.

[1]Recall: the *centralizer* of a subgroup H in a group K is $\mathbf{C}_K(H) := \{x \in K \mid xh = hx \text{ for all } h \in H\}$.

3. Let G be a group. An element $g \in G$ is called a *nongenerator* if, for any $X \subseteq G$, if $\langle X \cup \{g\} \rangle = G$, then $\langle X \rangle = G$. A subgroup M of G is *maximal* if $M \neq G$ and, for any subgroup H of G satisfying $H \neq G$, if $M \subseteq H$, then $H = M$.

Show that the set of nongenerators of G is equal to the intersection $\Phi(G)$ of all maximal subgroups of G. ($\Phi(G)$ is called the *Frattini subgroup* of G.)

4. Suppose G is a finite group having a unique maximal subgroup. Prove $|G|$ is a power of a prime.[2]

[2]Contrary to what I stated in class, this result is false if G is not required to be finite. See the course web page for an explanation and counter-example.