MAT 511 **HW #1** Name ______________________________

8/30/13, due Friday 9/6/13

25 points

1. (Isaacs' Problem 1) Let G be a group of functions from a set Ω to itself, with the product defined by composition of functions.

(a) Show, if G contains a one-to-one function, then G is a subgroup of the symmetric group S_Ω. (Do not assume Ω is finite.)

(b) Find an example with $|G| \geq 2$ such that G does not contain any one-to-one functions.

2. Let G be a group of permutations of a set Ω (under composition), i.e., G is a subgroup of S_Ω. Suppose that G is abelian, and that G *acts transitively* on Ω, that is, for very x and y in Ω there exists $f \in G$ such that $f(x) = y$. Prove that the action of G on Ω is *free*, that is, if $g(x) = x$ for some $x \in \Omega$, then $g = \mathrm{id}_\Omega$.

3. (a) Suppose H and K are subgroups of a group G, and $G = H \cup K$. Show that $H = G$ or $K = G$.

(b) Suppose H_α is a proper subgroup of G for each α, and $G = \bigcup_\alpha H_\alpha$. Suppose $xy = yx$ for every $x \in H_\alpha, y \in H_\beta$, with $\alpha \neq \beta$. Show that G is abelian.

Hint: Let $x \in H_\alpha$ and let $K = \{y \in G \mid xy = yx\}$. Observe that K is a subgroup of G. (K is the *centralizer* of x in G.) Apply part (a).