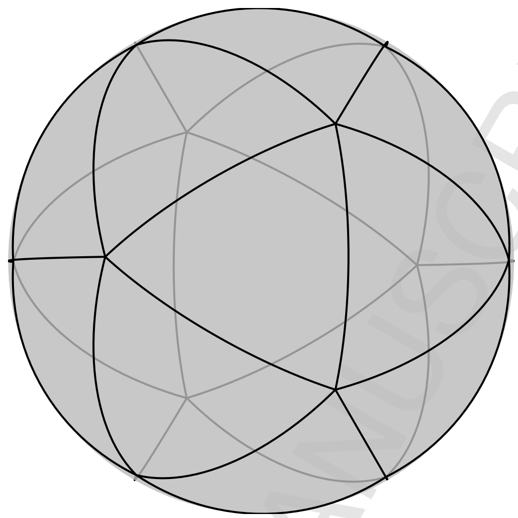

Figure 1: The icosahedron, projected radially onto its circumsphere.

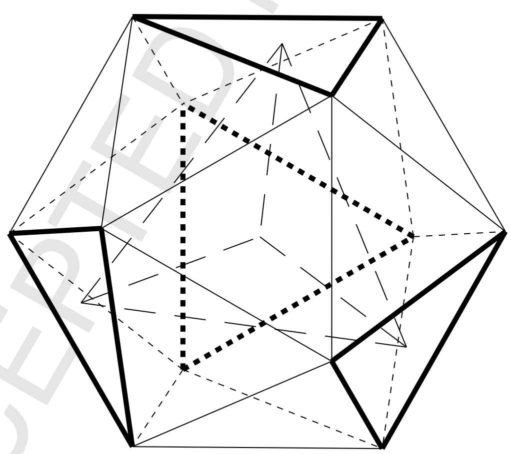

Figure 2: The icosahedron with inscribed tetrahedron.

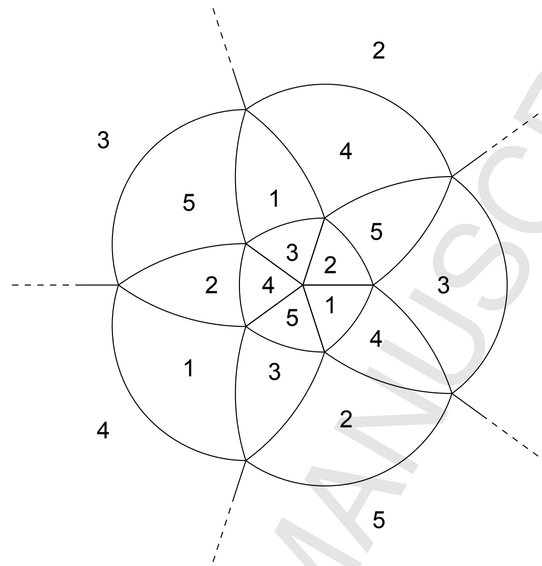

Figure 3: Tetrahedral face numbering of the icosahedron under stereographic projection (the outer radial lines meet at the vertex at infinity).

the icosahedron are labeled by a number $\nu \in \{1, \dots, 5\}$. Figure 3 exhibits such a numbering after stereographic projection.

The group Γ of rotations of the icosahedron acts transitively on the set of 20 faces with stabilizer of order 3 at each face and so has order 60. Γ also acts faithfully on the set of 5 tetrahedra as constructed above and so we obtain an embedding $\Gamma \hookrightarrow S_5$. Since A_5 is the only subgroup of S_5 of order 60 we must thus have:

$$\Gamma \simeq A_5$$

It will be useful later to have explicit generators for Γ. Thus let S be a rotation through $2\pi/5$ about the axis of symmetry joining the antipodal vertex pair $0, \infty$ and let T be the rotation through π about the axis of symmetry joining the midpoints of the antipodal edge pair $[0, \epsilon + \epsilon^{-1}], [\infty, \epsilon^2 + \epsilon^{-2}]$. Using the face numbering in figure 3, S, T correspond to the permutations:

$$S = (12345) \qquad T = (12)(34) \tag{2.2}$$