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ALGEBRAICALLY CLOSED GROUPS AND SOME EMBEDDING THEOREMS

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Abstract. Using the notion of algebraically closed groups, we obtain a new series of embedding theorems for R-groups and their generalizations R_π-groups. We show that any group of this type can be embedded in a simple exponential group. We also apply our method to determine the structure of R-groups whose non-trivial subgroups intersect non-trivially.

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The group $\mathbb{Z}_2$ is the only finite group which has just two conjugacy classes. Is there any infinite group with the same property? Denis Osin [6], proved that the answer is yes and in fact there is a finitely generated infinite group with exactly two conjugacy classes. His method is based on the small cancellation theory over relatively hyperbolic groups. Another example of such groups is obtained by Higman, Neumann and Neumann using their well-known embedding methods, [2]. In this article, we prove that any R-group can be embedded in a $\mathbb{Q}$-group which has only two conjugacy classes. We use this idea to prove that, if any two non-trivial subgroup of an R-group G have non-trivial intersection, then G is a subgroup of $\mathbb{Q}$. Remember that a group G is called R-group, if for any integer m, the equality $x^m = y^m$ implies $x = y$. We also give a generalization of these groups and define an R_π-group to be a group G such that the equality $x^m = y^m$ implies $x = y$, whenever m is a π'-number. We prove that any R_π-group can be embedded in a simple $\mathbb{Q}_\pi$-group, whose elements of the same orders are conjugate.

As an application of our method, we investigate the groups in which every two non-trivial subgroup have non-trivial intersection. Clearly such a group is torsion free or a p-group for some prime. The groups $\mathbb{Z}$, $\mathbb{Q}$ and $\mathbb{Z}_{p^\infty}$ are examples of such groups. It seems that the general case of this type of

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groups is hard to recognize. In this article we show that if such a group is also an R-group, then it can be embedded in $\mathbb{Q}$.

1. Algebraically closed groups

A group G is called *algebraically closed* (a.c. for short), if any finite consistent system of equations and inequations with coefficients from G has a solution in G. A system

$$S = \{w_i(\bar{g}, \bar{x}) = 1; (1 \leq i \leq r), w_j(\bar{g}, \bar{x}) \neq 1; (r+1 \leq j \leq s)\} \quad (I)$$

with coefficients $\bar{g}$ in G is called consistent, if there is a group K containing G, such that S has a solution in K. One can generalize this definition to an arbitrary class of groups: Let $\mathfrak{X}$ be a class of groups. A group $G \in \mathfrak{X}$ is called a.c. relative to $\mathfrak{X}$, if every $\mathfrak{X}$-consistent system S has a solution in G. Here, $\mathfrak{X}$-consistency means that there exists a group $K \in \mathfrak{X}$ which contains G and S has a solution in K.

The next lemma and theorem are proved in [7] for the class of all groups, but we give here the proofs again for the sake of completeness of this note. Remember that a class of groups is called *inductive*, if it contains the union of any chain its elements.

Lemma 1.1. *Let $\mathfrak{X}$ be an inductive class of groups which is closed under the operation of taking subgroups. Let $G \in \mathfrak{X}$. Then there is a group $H \in \mathfrak{X}$ with the following properties,*

1- G is a subgroup of H.
2- Every $\mathfrak{X}$-consistent system S of the from (I), has a solution in H.
3- $|H| = \max\{\aleph_0, |G|\}$.

Proof. We may assume that G is infinite, so let $|G| = \kappa$. Clearly the cardinality of the set of all systems of the form (I) is also κ. We suppose that this set is well-ordered as $\{S_\alpha\}_\alpha$. Let $G_0 = G$ and suppose that $G_\gamma \in \mathfrak{X}$ is already defined in such a way that $|G_\gamma| = \kappa$ and $\beta < \gamma$ implies $G_\beta \subseteq G_\gamma$. Let

$$K_\alpha = \bigcup_{\gamma<\alpha} G_\gamma.$$

Clearly, $K_\alpha \in \mathfrak{X}$ and $|K_\alpha| = \kappa$. If S_α is not $\mathfrak{X}$-consistent, then we set $G_\alpha = K_\alpha$, otherwise there is a $K \in \mathfrak{X}$ which contains K_α and S_α has a solution, say $\bar{u} = (u_1, \ldots, u_n)$ in K (n is the number of indeterminate in S_α). Let

$$G_\alpha = \langle K_\alpha, u_1, \ldots, u_n \rangle \leq K.$$

Then $G_\alpha \in \mathfrak{X}$ and $|G_\alpha| = \kappa$. So, for any $\alpha < \kappa$, we have defined a G_α. Note that, we have also

$$\beta < \alpha \Rightarrow G_\beta \subseteq G_\alpha.$$

Now, the group $H = \cup G_\alpha \in \mathfrak{X}$ has the required properties. □

Theorem 1.2. *Let $\mathfrak{X}$ be an inductive class of groups which is closed under the operation of taking subgroups. Let $G \in \mathfrak{X}$. Then, there exists a group $G^* \in \mathfrak{X}$, with the following properties,*

1- G is a subgroup of G^*.

2- G^* is a a.c. relative to $\mathfrak{X}$.

3- $|G^*| = \max\{\aleph_0, |G|\}$.

Proof. Let $G^0 = G$ and $G^1 = H$, where H is the group constructed in the lemma. Suppose G^m is already defined and G^{m+1} is the group which is proved to does exist for G^m in the lemma. Let $G^* = \cup G^m$. Therefore, $G^* \in \mathfrak{X}$, satisfies conditions 1 and 3. To prove 2, suppose S is a consistent system, with coefficients from G^*. Since S is finite, so there is an m such that all of the coefficients of S belong to G^m. So, S has a solution in $G^{m+1} \subseteq G^*$. □

As an application of this theorem, we prove that there are countable torsion free groups with exactly two conjugacy classes. Note that we can use a similar arguments to prove the existence of torsion free groups of any infinite cardinality with just two conjugacy classes. Also note that the type of the group we are giving here, is not new, it is known from works of Higman, Neumann and Neumann [2].

Corollary 1.3. *There exists a countable torsion free group with exactly two conjugacy classes.*

Proof. Suppose $\mathfrak{X}$ is the class of all torsion free groups. Hence $\mathfrak{X}$ is inductive and closed under the operation of taking subgroups. We, begin with the group $G = \mathbb{Z}$. Suppose $G^* \in \mathfrak{X}$ is the a. c. group relative to $\mathfrak{X}$, which is constructed for G in the theorem. We show that G^* is the required group. Let $a, b \in G^*$ be two non-identity elements. Consider the equation $xax^{-1} = b$. Let

$$G^*_{a,b} = \langle G^*, t : \ tat^{-1} = b \rangle$$

be an HNN-extension of G^*. We know that every torsion element of this HNN-extension is conjugate to a torsion element of G^*, so $G^*_{a,b}$ is torsion free. It also contains G^* as a subgroup and clearly t is a solution for $xax^{-1} = b$ in $G^*_{a,b}$. Therefor, there is already a solution in G^*. Hence G^* is a countable torsion free group with just two conjugacy classes. □

As we know from [1], the group G^* has many interesting properties: every $\mathfrak{X}$-group with a solvable word problem embeds in G^*, so G^* contains every torsion free hyperbolic group. However G^* is not finitely generated and even it has no recursive presentation.

2. Embedding theorems for R-groups

In this section we use the concept of algebraically closed groups to investigate the groups in which every two non-trivial subgroup have non-trivial intersection. Clearly such a group is torsion free or a p-group for some prime. The second case is very easy to investigate (Proposition 2.3 bellow). The groups $\mathbb{Z}$, $(\mathbb{Q}, +)$ and $\mathbb{Z}_{p^\infty}$ are examples of such groups. It is easy to see that all torsion free abelian group with this property can be embedded in $\mathbb{Q}$. Torsion free non-abelian groups with this property are constructed by Adian and Olshanskii, [5]. In [8], the author defined a topology-like structure over groups and he began to study the properties of groups equipped with such a *topo-systems*. Here we don't need to go on details of *topo-groups* and related concepts. But it will be very helpful, if we say that the problem of recognizing groups with non-trivial intersection of subgroups is appeared to the author, when he was working on the structure of *Hausdorff topo-groups* with respect to the *cofinite topo-system*.

Theorem 2.1. *Let G be an R-group in which any two non-trivial subgroups have non-trivial intersection. Then G can be embedded in $\mathbb{Q}$.*

Proof. Suppose $\mathfrak{X}$ is the class of all R-groups. Then $\mathfrak{X}$ is inductive and closed under subgroups. We have $G \in \mathfrak{X}$. So by the previous section there exits a group G^* which is algebraically closed in the class $\mathfrak{X}$ and it contains G. We prove that G^* has just two conjugacy classes. Let $a, b \in G^*$ be non-trivial. We consider the following HNN-extension

$$G^*_{a,b} = \langle G^*, t : tat^{-1} = b\rangle.$$

We know that $G^* \subseteq G^*_{a,b}$ and the equation $uau^{-1} = b$ has a solution in $G^*_{a,b}$. Therefore, if we prove that $G^*_{a,b} \in \mathfrak{X}$, then we will conclude that a and b are conjugate in G^*. Suppose $u, v \in G^*_{a,b}$ and $u^k = v^k$. We show that $u = v$. First, suppose u and v have cyclically reduced form

$$u = u_0 t^{\epsilon_1} u_1 t^{\epsilon_2} \dots t^{\epsilon_n} u_n$$

$$v = v_0 t^{\eta_1} v_1 t^{\eta_2} \dots t^{\eta_m} v_m,$$

where all $u_i, v_j \in G^*$ and $\epsilon_i, \eta_j = \pm 1$. Since, u and v are cyclically reduced and $u^k = v^k$, so using the lemma of Britton [2], we most have $m = n$ and all $\epsilon_i = \eta_i$. So, we have

$$v = v_0 t^{\epsilon_1} v_1 t^{\epsilon_2} \dots t^{\epsilon_n} v_n.$$

Now, we have

$$1 = u^k v^{-k} = u_0 t^{\epsilon_1} u_1 \dots t^{\epsilon_n} u_n \dots u_0 t^{\epsilon_1} u_1 \dots t^{\epsilon_n} u_n v_n^{-1} t^{-\epsilon_n} \dots t^{-\epsilon_1} v_0^{-1} \dots$$

If $m = n = 0$ then $u_0^k = v_0^k$ and since $G^* \in \mathfrak{X}$ we get $u = v$. The case $m = 0$ and $n \neq 0$ (or $m \neq 0$ and $n = 0$) is impossible again by the Britton's lemma. So, we assume that $m, n > 0$. The right hand side of the above equality most be reduced, so for example we have $\epsilon_n = 1$ and $u_n v_n^{-1} \in \langle a \rangle$. Hence there is a j such that

$$t^{\epsilon_n} u_n v_n^{-1} t^{-\epsilon_n} = b^j.$$

Therefore $t^{\epsilon_n} v_n = b^{-j} t^{\epsilon_n} u_n$. Hence

$$v = v_0 t^{\epsilon_1} v_1 \ldots t^{\epsilon_{n-1}} v_{n-1} b^{-j} t^{\epsilon_n} u_n.$$

Let $v'_{n-1} = v_{n-1} b^{-j}$. Then

$$v = v_0 t^{\epsilon_1} v_1 \ldots t^{\epsilon_{n-1}} v'_{n-1} t^{\epsilon_n} u_n.$$

Continuing this way, finally we get

$$v = v'_0 t^{\epsilon_1} u_1 \ldots t^{\epsilon_{n-1}} u_{n-1} t^{\epsilon_n} u_n.$$

This implies that $u^{k-1} = v^{k-1} v'_0 u_0^{-1}$. Expanding, we obtain

$$1 = u_0 t^{\epsilon_1} u_1 \ldots t^{\epsilon_n} u_n \ldots u_0 t^{\epsilon_1} u_1 \ldots t^{\epsilon_n} u_n u_0 (v'_0)^{-1} u_n^{-1} t^{-\epsilon_n} \ldots,$$

and hence, for example we obtain $\epsilon_n = 1$ and $u_n u_0 (v'_0)^{-1} u_n^{-1} = a^j$ for some j. Therefore $v'_0 u_0^{-1} = u_n^{-1} a^{-j} u_n$ and so

$$\begin{aligned} u^{k-1} &= v^{k-1} u_n^{-1} a^{-j} u_n \\ &= v^{k-2} v'_0 t^{\epsilon_1} u_1 \ldots t^{\epsilon_n} a^{-j} u_n. \end{aligned}$$

But, since $a^{-j} u_n \equiv u_n$ $(mod\ \langle a \rangle)$, so we get $u^{k-1} = v^{k-1}$ and therefore induction shows that $u = v$.

Now, suppose that u or v is not cyclically reduced. We know that

$$u = w_1 u' w_1^{-1}, \quad v = w_2 v' w_2^{-1},$$

where u' and v' are cyclically reduced. We have

$$(w_2^{-1} w_1) u'^k (w_2^{-1} w_1)^{-1} = v'^k.$$

If $w_1 = w_2$, then the result follows from the previous case. If $w_1 \neq w_2$, then the right side of the above equality is cyclically reduced while the left side is just reduced, which is impossible.

Hence we proved that $G^*_{a,b} \in \mathfrak{X}$, and so G^* has two conjugacy classes. We conclude that the group G^* is divisible, because for any $1 \neq u \in G^*$ and any natural number n, the elements u and u^n are conjugate, so there is z such that $u = z u^n z^{-1}$ and hence $u = (z u z^{-1})^n$. Now, in addition G^* is an R-group, so it is a $\mathbb{Q}$-group.

Now let $x, y \in G$ be non-trivial elements. Then $\langle x \rangle \cap \langle y \rangle \neq 1$, so there are non-zero integers p and q such that $x^p = y^q$. Therefore $y = x^{\frac{p}{q}}$ and hence if we fix x the result follows. □

Note that the group G^* obtained in the above proof is a $\mathbb{Q}$-group (it is divisible and R-group), it has just two conjugacy classes, and any R-group with solvable word problem embeds in G^*. The proof also shows that the next corollary is true.

Corollary 2.2. *Every R-group can be embedded is a $\mathbb{Q}$-group with two conjugacy classes.*

As we said in the introduction, a group with non-trivial intersection of non-trivial subgroup is torsion free or a p-group for some prime p. The next proposition shows that the second case can be investigate by a very elementary argument, and so the hardest part of the problem, is the case of torsion free groups which are not R-groups.

Proposition 2.3. *Let G be a p-group such that the intersection of any two non-trivial subgroups is non-trivial. Then G has a unique subgroup of order p. The converse is also true.*

Proof. Let G be a p-group and the intersection of any two non-trivial subgroup of G be non-trivial. Suppose x is a non-identity element of G. Then $\langle x\rangle$ has a subgroup A of order p. For any $1 \neq B \leq G$, we have $A \cap B \neq 1$, so $A \subseteq B$ and hence A is a unique subgroup of G of order p. Conversely let G has a unique subgroup of order p, say A. Then clearly for any non-trivial subgroup B, we have $A \subseteq B$, and hence every two non-trivial subgroups of G have non-trivial intersection. □

Note that the subgroup A in the proof the above proposition is *minimum* in the set of non-trivial normal subgroups of G. So, by a well-known theorem of Birkhoff, such a group G is sub-directly irreducible.

Corollary 2.4. *Any p-group with non-trivial intersection of non-trivial subgroups is sub-directly irreducible.*

The author encountered another class of groups, when he was working on the structure of groups which are Hausdorff with respect to *normal topo-system*. These are groups G, satisfying the condition

$$\langle x\rangle \cap \langle y\rangle = 1 \Rightarrow \langle x^G\rangle \cap \langle y^G\rangle = 1.$$

Theorem 2.5. *Let G be a group satisfying the condition*

$$\langle x\rangle \cap \langle y\rangle = 1 \Rightarrow \langle x^G\rangle \cap \langle y^G\rangle = 1.$$

Then for any $a, u \in G$, there are non-zero integers m and n such that $ux^n u^{-1} = x^m$. Further, if G is also an R-group, then there exists a $\mathbb{Q}$-group G^* and a family of subgroups $A_i \leq G^*$, such that

1- $G \subseteq \bigcup_i A_i$.
2- $A_i \cong \mathbb{Q}$.
3- $A_i \cap A_j = 1$, for $i \neq j$.

Proof. Let $A = G \setminus 1$ and define a binary relation on A by

$$x \equiv y \Leftrightarrow \langle x\rangle \cap \langle y\rangle \neq 1.$$

This is an equivalence relation on A. Let $E(x)$ be the equivalence class of x and $\{E_i : i \in I\}$ the set of all such classes. Then we have

$$G = \bigcup_i \bigcup_{y \in E_i} \langle y\rangle.$$

Now, since

$$\langle x\rangle \cap \langle y\rangle = 1 \Rightarrow \langle x^G\rangle \cap \langle y\rangle = 1,$$

so, $\langle x^G\rangle \subseteq \bigcup_{y\in E(x)}\langle y\rangle$. Let $u \in G$ be arbitrary. Then there is a $y \in E(x)$ such that $uxu^{-1} \in \langle y\rangle$. Hence, for some i, m and n we have

$$uxu^{-1} = y^i, \quad x^m = y^n.$$

Therefore, for any $x, u \in G$, there are non-zero integers m and n such that $ux^n u^{-1} = x^m$. Now, suppose G is R-group. As we saw before, there exists a $\mathbb{Q}$-group G^* containing G as a subgroup. Let $E_i = E(x)$. Then we have

$$\begin{aligned} E_i &= \{y \in G :\ \exists m, n \neq 0,\ x^m = y^n\} \\ &\subseteq \{x^\alpha :\ \alpha \in \mathbb{Q}\} \\ &\subseteq G^*. \end{aligned}$$

Suppose $A_i = \{x^\alpha :\ \alpha \in \mathbb{Q}\}$. Then any A_i is a subgroup of G^* and we have 1-2-3. □

3. One more embedding theorem

In this section, we generalize the embedding theorem of R-groups proved in the previous section to more general classes of groups. Let π be a set of prime numbers. We consider the ring

$$\mathbb{Q}_\pi = \{\frac{m}{n} \in \mathbb{Q} :\ n \ is\ a\ \pi' - number\}.$$

If π consists of a single element p, then we denote this ring by $\mathbb{Q}_p$. A group G is an R_π-group, iff for any x and y and any π'-number n, we have the implication

$$x^n = y^n \Rightarrow x = y.$$

The order of any element of such a group is infinite or a π-number. We say that G is π-divisible, iff for any $x \in G$ and any π'-number n, there exists $y \in G$, such that $y^n = x$. Note that if a group G is both R_π-group and π-divisible, then there is a unique y satisfying $y^n = x$, for a given π'-number n. So, we denote this element by $x^{\frac{1}{n}}$. Now, a group which is both R_π-group and π-divisible, can be regarded as an exponential group over the ring $\mathbb{Q}_\pi$ (or $\mathbb{Q}_\pi$-group for short) via $x^{\frac{m}{n}} = (x^{\frac{1}{n}})^m$. The reader most consult [3] and [4] for the theory of exponential groups. Note that any $\mathbb{Q}_\pi$-group is also R_π-group and π-divisible. We are now ready to prove the main theorem of this section.

Theorem 3.1. *Let G be an R_π-group. Then there exists a $\mathbb{Q}_\pi$-group G^* containing G such that*

1- G^* is simple,

2- element of the same order in G^* are conjugate,

3- G^* is not finitely generated,

4- and every finite π-group embeds in G^*.

Proof. The proof is almost the same as 2.1 and [2]. Let $\mathfrak{X}_\pi$ be the class of all R_π-groups. This class is also inductive and closed under subgroup. Since $G \in \mathfrak{X}_\pi$, so there exists a group $G^* \in \mathfrak{X}_\pi$ containing G, which is algebraically closed in the class $\mathfrak{X}_\pi$. Suppose $1 \neq a, b \in G^*$ have the same order. We show that a and b are conjugate. Suppose the HNN-extension

$$G^*_{a,b} = \langle G^*, t : tat^{-1} = b \rangle.$$

It is enough to show that $G^*_{a,b}$ belongs to $\mathfrak{X}_\pi$. But the proof of this part is similar to the proof of 2.1 (except in the induction step, in this case we don't argue by induction, since $k-1$ may not be a π'-number. Instead we continue to reduce $k-1$ to $k-2$ and so on to get $u = v$). This shows that elements of the same order in G^* are conjugate. To show that G^* is a $\mathbb{Q}_\pi$-group, let $x \in G^*$ and n be a π'-number. We know that the orders of x and x^n are equal. Hence there is a $z \in G^*$ such that $x = zx^n z^{-1}$. Suppose $u = zxz^{-1}$. Then $u^n = x$ and hence G^* is $\mathbb{Q}_\pi$-group.

We show that G^* is simple. Note that $G^* * \langle x \rangle$ is an R_π-group. This follows from the fact that G^* is an R_π-group and the uniqueness of the reduced form of elements in free products. Let $1 \neq a, w \in G^*$ be arbitrary elements and suppose $u = wxw^{-1}x^{-1}$ and $v = axw^{-1}x^{-1}$. Then u and v are reduced in the free product and so they have infinite orders. Hence we can consider the HNN-extension

$$M = \langle G^* * \langle x \rangle, t : tut^{-1} = v \rangle.$$

With the same argument as for G^*, we see that M is also an R_π-group. The equation

$$twxw^{-1}x^{-1}t^{-1} = axw^{-1}x^{-1}$$

has a solution for t and x in M, therefore it has already a solution in G^*. We have

$$a = (twt^{-1})(txw^{-1}x^{-1}t^{-1})(xw^{-1}x^{-1}),$$

so $a \in \langle w^{G^*} \rangle$. Hence, for all $1 \neq w \in G^*$, we have $G^* = \langle w^{G^*} \rangle$. This proves that G^* is simple.

Now, since G^* is non-abelian simple group, we have $Z(G^*) = 1$. On the other hand, for any finite subset $a_1, \ldots, a_m \in G^*$, the system

$$a_1 x = x a_1, \ldots, a_m x = x a_m, x \neq 1,$$

has a solution in the R_π-group $G^* \times \langle x \rangle$, so it has a solution in G^*. Therefore

$$C_{G^*}(\langle a_1, \ldots, a_m \rangle) \neq 1.$$

This proves that G^* is not finitely generated.

Finally, suppose that H is a finite π-group and

$$H = \{1 = g_0, g_1, \ldots, g_m\}.$$

For any $1 \leq i, j \leq m$ there exists a unique $0 \leq k(i,j) \leq m$, such that $g_i g_j = g_{k(i,j)}$. The group $G^* \times H$ is clearly an R_π-group and the system

$$x_i x_j = x_{k(i.j)}, x_1 \neq 1, \ldots, x_m \neq 1$$

has a solution in $G^* \times H$ and hence it has a solution in G^*. This shows that H is embedded in G^*.

$\square$

Corollary 3.2. *Suppose G is an R_p-group such that non-identity elements of G have order p. Then G can be embedded in a $\mathbb{Q}_p$-group which has just two conjugacy classes. This group is not finitely generated and any finite p-group embeds in it.*

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