MAT 441 **Exam 3** Name ____________________________
4/17/09
70 points

Provide some justification for all nontrivial claims, to receive full credit. You may use any theorems stated in lecture without proof.

1.(20) (a) Show that every integer n less than or equal to 2 occurs as the Euler-Poincaré characteristic of some compact surface without boundary.

(b) Up to homeomorphism, how many different compact surfaces S are there with given Euler-Poincare characteristic $\chi(S)$, in case

(i) $\chi(S)$ is even.

(ii) $\chi(S)$ is odd.

(c) Make a list of all compact surfaces with non-negative Euler-Poincaré characteristic.

2.(20) Each expression below is meant to identify the identification pattern for a planar identification diagram for a topological space. In each case

(i) determine whether the quotient space is a topological surface (without boundary);

(ii) if the quotient space is a surface, determine whether the that surface is orientable or not.

(iii) compute the Euler-Poincaré characteristic of the quotient space (whether or not the quotient space is a surface).

(iv) if the quotient space is a surface, identify it according to the classification theorem for surfaces (i.e., as a connected sum of $\mathbb{T}^2$'s and/or $\mathbb{P}^2$'s. Hint: Use (iii)

(a) $abcc^{-1}b^{-1}a^{-1}b$

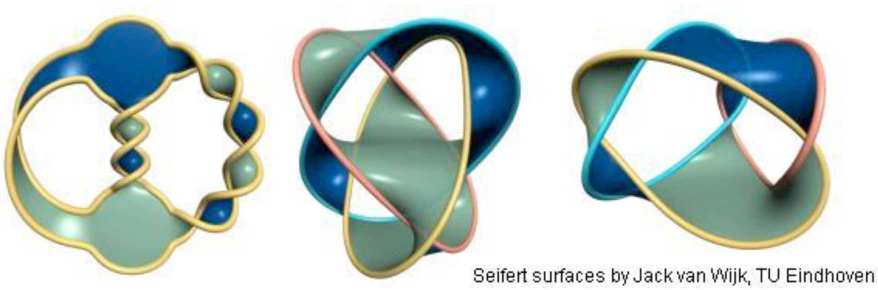
(b) $abcba^{-1}c^{-1}$

(c) $abcdc^{-1}d^{-1}a^{-1}b^{-1}$

3.(15) (a) Suppose S' is a compact surface with boundary, obtained from a compact surface S by removing k open disks (with disjoint closures)[1]. Express $\chi(S')$ in terms of $\chi(S)$ and k.

(b) Make a list of all compact surfaces with boundary having non-negative Euler-Poincare characteristic.

(c) Identify the surfaces pictured below (from the Math/Stat Department website `http://www.cefns.nau.edu/Academic/Math/researchInterests/`) by calculating the Euler-Poincaré characteristic and counting the number of boundary components. Construct a cell decomposition by inserting vertices along the boundary components and 1-cells in the surface separating the twists or crossings. These surfaces are orientable - if you look at the colored pictures on the web page you'll see that the surfaces have two sides, one green and one blue.



[1]It can be shown that every compact surface with boundary can be obtained this way.

4.(5) Let V be the set of nonempty subsets of $\{1, 2, 3\}$. Let K be the abstract simplicial complex with vertex set V whose simplices are the subsets σ of V which are *linearly ordered,* that is, for every $A, B \in \sigma$, $A \subseteq B$ or $B \subseteq A$. Show that K determines a triangulation of the standard simplex Δ^2.

5.(10) Let K be the abstract simplicial complex $K = \{1, 2, 3, 4, 12, 13, 23, 24, 34, 123\}$[2]. Compute the simplicial homology of K (with real coefficients).

[2] We are using the standard shorthand, denoting sets of numbers by listing their elements without commas or set braces.