MAT 441C   **HW #6**   Name ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

4/1/13 (due Friday 4/5/13)

25 points   Justify all answers. Unsupported answers may not receive full credit.

1. Let X and Y be topological spaces, and suppose Y is compact.

    (a) Let $x_0 \in X$, and let W be an open subset of $X \times Y$ containing $\{x_0\} \times Y$. Show that there exists an open subset V of X such that $\{x_0\} \times Y \subseteq V \times Y \subseteq W$.

    Hint: Cover $\{x_0\} \times Y$ with basic (product) open sets.

    (b) Let $A \subseteq X$ and suppose W is an open subset of $X \times Y$ containing $A \times Y$. Use (a) to show that there is an open subset V of X such that $A \subseteq V$ and $V \times Y \subseteq W$.

    (c) Suppose X and Y are both compact. Prove $X \times Y$ is compact.

    Hint: Given an open cover of $X \times Y$, extract a finite subcover of $\{x\} \times Y$ for each x. Then use part (a) to construct an open cover of X.

    Extract a finite subcover, and use the corresponding finite set of points from X to find a finite subcover of the original cover.

2. (a) Show, by cutting and pasting planar diagrams, that the surface-with-boundary given by the diagram with boundary word aab is homoemorphic to the Möbius band.

(b) In a similar fashion, show the surface given by the diagram with boundary word $aabb$ is homeomorphic to the Klein bottle.

(c) Similarly, show the diagram with boundary word $abcda^{-1}b^{-1}c^{-1}d^{-1}$ is homeomorphic to $T^2 \, \sharp \, T^2$,

(d) Suppose a diagram for a compact surface or surface-with-boundary has boundary word with a letter appearing twice with the same exponent. Prove the surface has a closed subset homeomorphic to the Möbius band.