MAT 441C **HW #5** Name ____________________

3/7/13 (due Monday 3/11 at 5:30 pm)

25 points

Justify all answers. Unsupported answers may not receive full credit.

1. Let $X = \{-1, 1\} \times [-1, 1]$. (So X is a union of two disjoint line segments.) Let $\sim$ the the equivalence relation on X defined by $(-1, y) \sim (1, y)$ for $-1 \leq y < 0$ (and $(x, y) \sim (x, y)$ for all $(x, y) \in X$). Show that the quotient space $X/\sim$ is not Hausdorff.

2. Determine, if possible, which of the following spaces are homeomorphic. Give a short justification for your answers. (Some of these spaces are not distinguishable using connectedness and/or compactness arguments.)

$$[0,1],\ (0,1],\ (0,\infty),\ [0,\infty),\ \mathbb{R},\ \mathbb{R}-\{0\},\ S^1,\ S^1-\{(1,0)\},\ \mathbb{R}^2,\ \mathbb{R}^2-\{(0,0)\},\ S^2,\ S^2-\{(0,0,1)\}$$

3. A space X is *countably compact* if every countable open cover of X has a finite subcover. A space is *sequentially compact* if every infinite sequence has a convergent subsequence.

(a) Prove: if X is countably compact, then every countably infinite subset of X has a limit point.

(b) Prove: if X is countably compact and first-countable, then every sequence in X has a convergent subsequence.

3. (15) A map $f : X \to Y$ is *locally constant* if every point of X has a neighborhood on which f is constant. (That is, $\forall x \in X, \exists$ open $U(x)$ such that $f(y) = f(x)$ for all $y \in U$.)

(a) Prove that any locally constant function is continuous.

(b) Prove that any locally constant function on a connected space X is constant.