MAT 441C **HW #1** Name ____________________

1/18/13 (due Friday 1/25/13)

25 points

Justify all answers. Unsupported answers may not receive full credit.

1. Let $X \subseteq \mathbb{R}^n$. Show that the open ϵ-ball $B_X(x, \epsilon)$ is an open subset of X, for any $x \in X$ and any $\epsilon > 0$.

2. A topological space is a set X endowed with a specified collection of subsets called "open sets," satisfying a few simple axioms. A subset K of X is then defined to be *closed* if and only if the complement $X - K$ is an open set. If $A \subseteq X$, the *closure* of A is the set $\bar{A}$ defined by

$$\bar{A} = \bigcap \{K \subseteq X \mid A \subseteq K \text{ and } K \text{ is closed}\}.$$

(a) Prove (i) $A \subseteq \bar{A}$, and (ii) if L is a closed subset of X and $A \subseteq L$, then $\bar{A} \subseteq L$.

(b) Prove: for all $x \in X$, $x \in \bar{A}$ if and only if, for any open subset U of X, if $x \in U$, then $U \cap A \neq \emptyset$. For one of the two implications, you will need to use the fact that $\bar{A}$ is a closed set, which will be proven in class.

3. Let X be a finite set. A *topology* on X is a collection $\mathcal{T}$ of subsets of X that includes $\emptyset$ and X and is closed under pairwise intersections and unions. In this case the pair $(X, \mathcal{T})$ is called a *finite topological space.* A *homeomorphism* of finite topological spaces $(X, \mathcal{T}_X)$ and $(Y, \mathcal{T}_Y)$ is a bijection $h\colon X \to Y$ with the property that, for every subset U of X, $U \in \mathcal{T}_X$ if and only if $h(U) \in \mathcal{T}_Y$. This relation is reflexive, symmetric, and transitive.

There are 29 distinct topologies on the set $X = \{1, 2, 3\}$. List them all, and sort them into homeomorphism classes. (It's okay to write, for instance, 12 instead of $\{1, 2\}$.)