Show work or otherwise justify your answers. Unsupported answers (i.e. calculator output) will not receive full credit. You may check your answers with a calculator or computer.

1.)(a) Determine all values of c for which the constant function y(t) = c a solution of the equation

$$\frac{dy}{dt} = 4 - y^2.$$

(b) For what range of *y*-values are solutions monotonic increasing?

: Hint: y(t) is monotonic increasing when $\frac{dy}{dt}$ is positive.

(c) Sketch the direction field for this equation, and a few solutions.¹ Determine the limiting behavior of solutions as $t \to \infty$, for all possible initial values of $y_0 = y(0)$.

 $^{^1 {\}rm You}$ can use JOde.

2. (a) Find the general solution y(t) of the equation from Problem 1.

Hint: After some manipulation, integrate by using a partial fractions decomposition - be sure to factor the denominator first.

(b) Find the particular solution y(t) with initial value y(0) = 1.

3. A linear first order ODE of the form $\frac{dy}{dt} = at + by$, where a and b are constants, always has a solution y(t) which is a linear function. Find such a solution for the equation

$$\frac{dy}{dt} = 2t + 3y,$$

by writing y(t) = mt + b and finding values for m and b so that y(t) satisfies the equation².

 $^{^{2}}$ You can check your answer by using JOde to plot solutions; you will see the linear solution as a major feature of the diagram.