Show work or otherwise justify your answers. Unsupported answers (i.e. calculator output) will not receive full credit. You may check your answers with a calculator or computer.

1. Determine whether or not the improper integral converges. If convergent, find the value.

(a)
$$\int_{2}^{\infty} \frac{1}{x^2 - 1} dx$$

(b)
$$\int_0^1 \frac{1}{x^2 - 1} \, dx$$

2. Find the length of the curve $y = \ln(\cos(x)), \ 0 \le x \le \frac{\pi}{3}$.

3. (a) Find the centroid of the region R described by the system of inequalities $x^2 \le y \le 4, x \ge 0$.

(b) Use your answer to (a), and Pappus' Theorem, to find the volume of the solids obtained by rotating the region R about (i) the y-axis, and (ii) the x-axis.

(c) Use Pappus' Theorem, and the formula for the volume of a sphere, to find the centroid of the quarter circle given by $x^2 + y^2 \le r^2$, $x \ge 0$, $y \ge 0$ without integrating.