MAT 137 **HW #8** Name SOLUTIONS

04/23/09 (due Wednesday 04/29/09)

10 points

Show work or otherwise justify your answers. Unsupported answers (i.e. calculator output) will not receive full credit. You may check your answers with a calculator or computer.

1. Let $\mathbf{v} = 2\mathbf{i} - \mathbf{j}$ and $\mathbf{w} = \mathbf{i} + 3\mathbf{j}$.

   (a) Find $3\mathbf{v} + \mathbf{w}$.

   $$3(2\hat{i} - \hat{j}) + (\hat{i} + 3\hat{j}) = 6\hat{i} - 3\hat{j} + \hat{i} + 3\hat{j} = \boxed{7\hat{i}}$$

   or $(7, 0)$

   (b) Find the length $\|\mathbf{v}\|$ of the vector $\mathbf{v}$.

   $$\|\vec{v}\| = \|2\hat{i} - \hat{j}\| = \sqrt{2^2 + (-1)^2} = \boxed{\sqrt{5}}$$

   (c) Find a vector parallel to $\mathbf{v}$ whose length is equal to 2.

   Hint: First find a unit vector parallel to $\mathbf{v}$, then multiply by 2.

   $$w = 2\left(\frac{\vec{v}}{\|\vec{v}\|}\right) = 2\left(\frac{2\hat{i} - \hat{j}}{\sqrt{5}}\right) = 2\left(\frac{2}{\sqrt{5}}\hat{i} - \frac{1}{\sqrt{5}}\hat{j}\right)$$

   $$= \boxed{\frac{4}{\sqrt{5}}\hat{i} - \frac{2}{\sqrt{5}}\hat{j}}$$

   (d) Compute the scalar (or dot) product $\mathbf{v} \cdot \mathbf{w}$.

   $$\vec{v} \cdot \vec{w} = (2\hat{i} - \hat{j}) \cdot (\hat{i} + 3\hat{j})$$

   $$= (2)(1) + (-1)(3) = 2 - 3 = \boxed{-1}$$

   (e) Use the scalar (or dot) product to find the cosine of the angle between $\mathbf{v}$ and $\mathbf{w}$.

   $$\cos(\theta) = \frac{\vec{v} \cdot \vec{w}}{\|\vec{v}\| \|\vec{w}\|} = \frac{-1}{\sqrt{5}\sqrt{1^2 + 3^2}} = \boxed{\frac{-1}{\sqrt{50}}}$$

   (f) Which vector below is perpendicular to $\mathbf{w} = \mathbf{i} + 3\mathbf{j}$? Circle the correct answer.

   Hint: Use dot product.

   $$\mathbf{2i + 3j}, \quad \mathbf{5i - 2j}, \quad \boxed{\mathbf{-6i + 2j}}, \quad \mathbf{i - 3j}$$

   $$(-6\hat{i} + 2\hat{j}) \cdot (\hat{i} + 3\hat{j}) = (-6)(1) + (2)(3) = 0$$

2. Find the scalar and vector projections of the vector $\mathbf{v} = \langle 5, 1, 2 \rangle$ onto the vector $\mathbf{w} = \langle 1, 1, 1 \rangle$.

   $$\text{scalar projection} = \vec{v} \cdot \left(\frac{\vec{w}}{\|\vec{w}\|}\right) = \frac{\vec{v} \cdot \vec{w}}{\|\vec{w}\|} = \boxed{\frac{-1}{\sqrt{10}}}$$

   $$\text{vector projection} = \frac{\vec{v} \cdot \vec{w}}{\vec{w} \cdot \vec{w}} \vec{w} = \left(\frac{-1}{10}\right)(\hat{i} + 3\hat{j}) = \boxed{-\frac{1}{10}\hat{i} + \frac{3}{10}\hat{j}}$$

3. (a) Find the vector (cross) product $\mathbf{v} \times \mathbf{w}$ of the vectors $\mathbf{v} = \langle 4, -2, 1 \rangle$ and $\mathbf{w} = \langle 1, 5, -2 \rangle$.

$$\vec{v} \times \vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -2 & 1 \\ 1 & 5 & -2 \end{vmatrix} = \left((-2)(-2) - 1\cdot 5,\ -((4)(-2) - 1\cdot 1),\ 4\cdot 5 - (1)(-2) \right)$$

$$= \boxed{(-1, 9, 22)}$$

(b) Find the area of the parallelogram spanned by $\mathbf{v}$ and $\mathbf{w}$.

Hint: The area is equal to the magnitude of the cross product.

$$\text{area} = \|\vec{v} \times \vec{w}\| = \|(-1, 9, 22)\| = \sqrt{(-1)^2 + 9^2 + 22^2}$$

$$= \boxed{\sqrt{566} \doteq 23.77}$$

4. (a) Write an equation for the plane perpendicular to the vector $\langle 3, -2, 5 \rangle$ which passes through the point $(1, 1, 1)$.

(b) Write an equation for the plane through the origin which contains the two vectors $\langle 1, 0, 2 \rangle$ and $\langle 0, 1, -4 \rangle$.

Hint: Use cross product to find a normal vector.

Omit

5. Write vector and scalar parametric equations describing the line which is perpendicular to the plane $10x - y + 5z = 10$ and passes through the point $(3, 1, 1)$.