MAT 137 **HW #2** Name SOLUTIONS

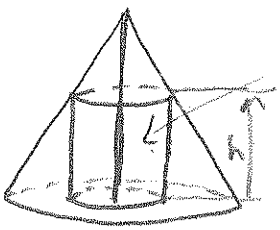
1/30/09 (due Wednesday 2/4/09)

10 points

Show work or otherwise justify your answers. Unsupported answers (i.e. calculator output) will not receive full credit. You may check your answers with a calculator or computer.

1. (a) Set up (but do not evaluate) an integral expression for the mass of a cone of radius 3 cm and height 6 cm, if the density $x$ centimeters from the axis of the cone is $x+2$ grams per cubic centimeter.

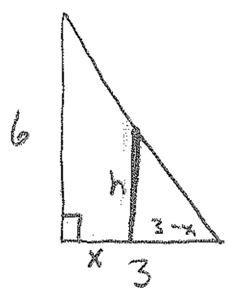
Hint: Use cylindrical shells.




density $= x+2$ $g/cm^3$

volume $= (\text{area}) \cdot dx$
$= 2\pi x h \, dx$

mass of cylindrical shell $\approx 2\pi x h (x+2) dx$




$$\frac{h}{6} = \frac{3-x}{3}$$

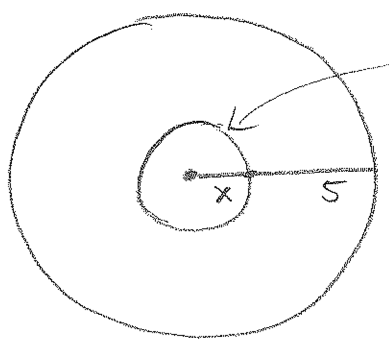
$$\Rightarrow h = 2(3-x) = 6-2x$$

Total mass =

$$\boxed{\int_0^3 2\pi x (6-2x)(x+2)\, dx}$$

(b (a) Set up (but do not evaluate) an integral expression for the mass of a planar disk of radius 5 cm if the density $x$ centimeters from the center of the disk is $1+x$ grams per square centimeter.

Hint: Use concentric circles.




density $= (1+x)$ $g/cm^2$

area of thin ring $\approx 2\pi x \, dx$

mass of thin ring $= 2\pi x (1+x) dx$

total mass $= \boxed{\int_0^5 2\pi x (1+x)\, dx}$

2. Find these integrals by making the indicated change of variables.

(a) $\int_0^2 \frac{x}{x^2+1}\,dx, \quad u = x^2+1$

$u = x^2+1 \qquad x=0 \Longrightarrow u=1$

$du = 2x\,dx \qquad x=2 \Longrightarrow u=5$

$$= \frac{1}{2}\int_1^5 \frac{1}{u}\,du = \frac{1}{2}\ln(u)\Big|_1^5 = \boxed{\frac{1}{2}\ln(5)}$$

(b) $\int_0^1 \frac{x}{\sqrt{2x+1}}\,dx, \quad u = 2x+1 \longrightarrow u-1 = 2x, \quad x = \frac{1}{2}(u-1)$

$dx = \frac{1}{2}\,du$

$x=0 \longrightarrow u=1$

$x=1 \longrightarrow u=3$

$$= \int_1^3 \frac{\frac{1}{2}(u-1)}{\sqrt{u}}\cdot\frac{1}{2}\,du$$

$$= \frac{1}{4}\int_1^3 \left(u^{1/2} - u^{-1/2}\right)du = \frac{1}{4}\left[\frac{2}{3}u^{3/2} - 2u^{1/2}\right]_1^3 = \boxed{\frac{\sqrt{3}}{6}}$$

3. Integrate by parts. (a) $\int x^2\sin(x)\,dx$

$u = x^2 \qquad du = 2x\,dx$

$dv = \sin(x)\,dx \qquad v = -\cos(x)$

$$= -x^2\cos(x) + \int 2x\cos(x)\,dx$$

$u = 2x \quad du = 2dx$

$dv = \cos(x)\,dx$

$v = \sin(x)$

$$= -x^2\cos(x) + 2x\sin(x) - \int 2\sin(x)\,dx$$

$$= \boxed{-x^2\cos(x) + 2x\sin(x) + 2\cos(x) + C}$$

(b) $\int x^2\ln(x)\,dx$

$u = \ln(x) \qquad du = \frac{1}{x}\,dx$

$dv = x^2\,dx \qquad v = \frac{1}{3}x^3$

$$\int x^2\ln(x)\,dx = \frac{1}{3}x^3\ln(x) - \int\left(\frac{1}{3}x^3\right)\left(\frac{1}{x}\right)dx$$

$$= \frac{1}{3}x^3\ln(x) - \frac{1}{3}\int x^2\,dx$$

$$= \boxed{\frac{1}{3}x^3\ln(x) - \frac{1}{9}x^3 + C.}$$