MAT 137 **HW #1** Name SOLUTIONS

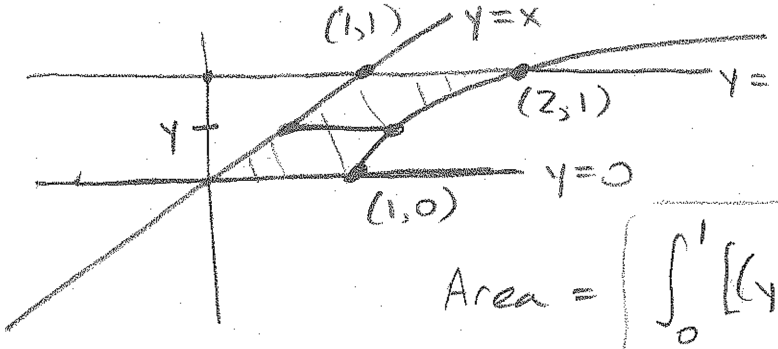
1/22/09 (due Wednesday 1/28/09)

10 points

Show work or otherwise justify your answers. Unsupported answers (i.e. calculator output) will not receive full credit. You may check your answers with a calculator or computer.

1. Let R be the region bounded by $y=0$, $y=1$, $y=x$, and $y=\sqrt{x-1}$. Set up, but *do not evaluate*, an integral or a sum of integrals, for the area of R, using

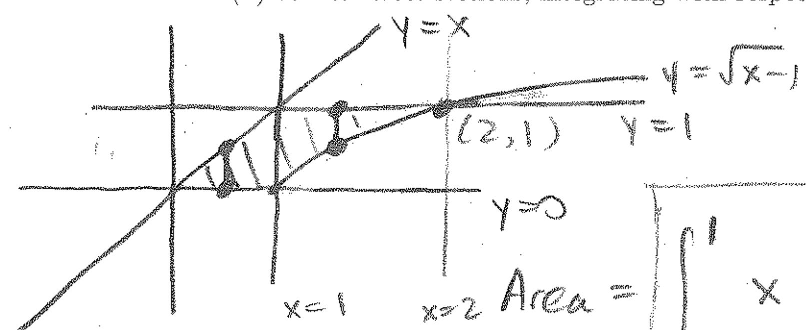
(a) horizontal sections, integrating with respect to y.



$y=\sqrt{x-1} \iff x = y^2+1$

$$\text{Area} = \boxed{\int_0^1 \left[(y^2+1) - y\right] dy}$$

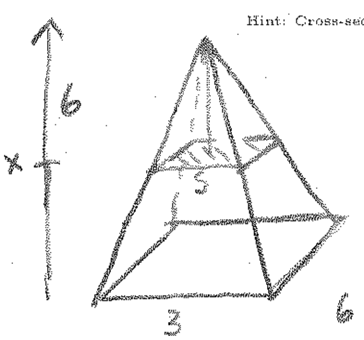
(b) vertical cross sections, integrating with respect to x.



$$\text{Area} = \boxed{\int_0^1 x\, dx + \int_1^2 \left(1 - \sqrt{x-1}\right) dx}$$

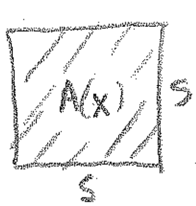
2. (a) Set up (but do not evaluate) an integral expression for the volume of a pyramid of height 6 cm, with base a square of side length 3 cm.

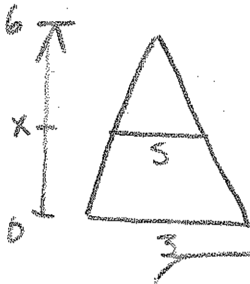
Hint: Cross-sections parallel to the base are squares.



$$\text{Volume} = \int_0^6 A(x)\, dx$$

$$A(x) = s^2$$





$\frac{3}{6} = \frac{s}{6-x}$ by similar triangles

$s = \frac{1}{2}(6-x)$, so

(continued on back)

$$\text{Volume} = \boxed{\int_0^6 \left[\tfrac{1}{2}(6-x)\right]^2 dx}$$

(b) Suppose the density of the pyramid in part (a) at points x centimeters above the base is $100 - x^2$ grams per cubic centimeter. Set up (but do not evaluate) an integral expression for the total mass of the pyramid.

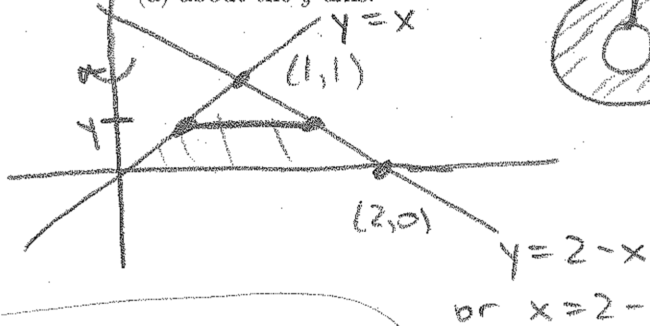
Note: If the density of an object (such as a horizontal section of the pyramid) is constant, then the mass is equal to density times volume.

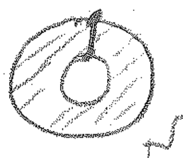
The mass of a horizontal (square) section at height x is equal to $(100 - x^2) \cdot A(x)\,dx$, so the total mass is

$$\boxed{\int_0^6 (100 - x^2)\left(\tfrac{1}{2}(6-x)\right)^2 dx}$$

3. Set up (but do not evaluate) an integral expression for the volume the solid obtained by rotating the region bounded by $y = x$, $y = 2 - x$, and the x axis

(a) about the y axis.





washers

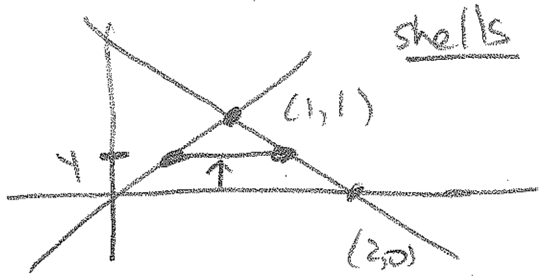
$$\boxed{V = \int_0^1 \pi\left[(2-y)^2 - y^2\right] dy}$$

Alternate answer (by shells)

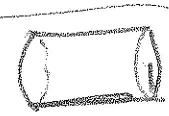
$$V = \int_0^1 2\pi x^2\,dx + \int_1^2 2\pi x(2-x)\,dx$$

(b) about the x-axis.

Hint: Use cylindrical shells.



shells



radius $= y$

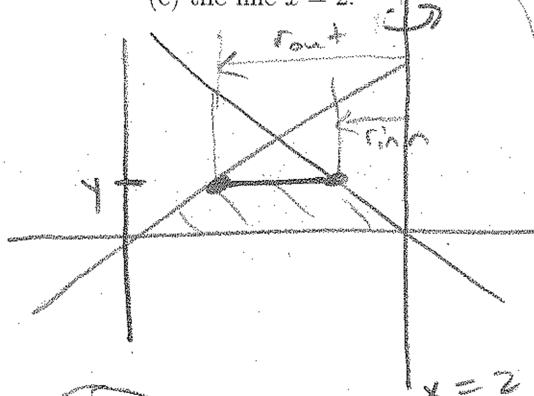
height $= (2-y) - y$

$$\boxed{V = \int_0^1 2\pi y\left[(2-y) - y\right] dy}$$

Alternate answer (by discs)

$$V = \int_0^1 \pi x^2\,dx + \int_1^2 \pi(2-x)^2\,dx$$

(c) the line $x = 2$.



$$\boxed{V = \int_0^1 \pi\left[(2-y)^2 - y^2\right] dy}$$

(same as (a), by symmetry)!

washers, $r_{out} = 2 - y$

$r_{in} = 2 - (2-y) = y$