

### WeBWorK assignment number Exercises\_7

This is an exercise assignment, treated differently from ordinary WeBWork assignments. The WeBWork due date is the same as the opening date, so that answers are available to students immediately. Students are to print the assignment and work the problems on paper. That work will be collected periodically and graded for effort. Exercises are worth 20 points (4% of the overall grade).

#### 1. (1 pt) Library/maCalcDB/setSeries6CompTests/benny\_ser1.pg

Test each of the following series for convergence by either the Comparison Test or the Limit Comparison Test. If either test can be applied to the series, enter CONV if it converges or DIV if it diverges. If neither test can be applied to the series, enter NA. (Note: this means that even if you know a given series converges by some other test, but the comparison tests cannot be applied to it, then you must enter NA rather than CONV.)

1.  $\sum_{n=1}^{\infty} \frac{2n^2}{n^3 + 8}$
2.  $\sum_{n=1}^{\infty} \frac{(-1)^n}{4n}$
3.  $\sum_{n=1}^{\infty} \frac{(\ln(n))^2}{n+3}$
4.  $\sum_{n=1}^{\infty} \frac{\cos^2(n)\sqrt{n}}{n^2}$
5.  $\sum_{n=1}^{\infty} \frac{2n^2}{n^7 + 8}$

#### 2. (1 pt) Library/maCalcDB/setSeries6CompTests/benny\_ser3.pg

Each of the following statements is an attempt to show that a given series is convergent or divergent not using the Comparison Test (NOT the Limit Comparison Test.) For each statement, enter C (for "correct") if the argument is valid, or enter I (for "incorrect") if any part of the argument is flawed. (Note: if the conclusion is true but the argument that led to it was wrong, you must enter I.)

1. For all  $n > 1$ ,  $\frac{1}{n \ln(n)} < \frac{2}{n}$ , and the series  $2 \sum \frac{1}{n}$  diverges, so by the Comparison Test, the series  $\sum \frac{1}{n \ln(n)}$  diverges.
2. For all  $n > 2$ ,  $\frac{n}{n^3-1} < \frac{2}{n^2}$ , and the series  $2 \sum \frac{1}{n^2}$  converges, so by the Comparison Test, the series  $\sum \frac{n}{n^3-1}$  converges.
3. For all  $n > 1$ ,  $\frac{\arctan(n)}{n^3} < \frac{\pi}{2n^3}$ , and the series  $\frac{\pi}{2} \sum \frac{1}{n^3}$  converges, so by the Comparison Test, the series  $\sum \frac{\arctan(n)}{n^3}$  converges.
4. For all  $n > 1$ ,  $\frac{n}{7-n^3} < \frac{1}{n^2}$ , and the series  $\sum \frac{1}{n^2}$  converges, so by the Comparison Test, the series  $\sum \frac{n}{7-n^3}$  converges.
5. For all  $n > 1$ ,  $\frac{\ln(n)}{n^2} < \frac{1}{n^{1.5}}$ , and the series  $\sum \frac{1}{n^{1.5}}$  converges, so by the Comparison Test, the series  $\sum \frac{\ln(n)}{n^2}$  converges.

6. For all  $n > 2$ ,  $\frac{\ln(n)}{n^2} > \frac{1}{n^2}$ , and the series  $\sum \frac{1}{n^2}$  converges, so by the Comparison Test, the series  $\sum \frac{\ln(n)}{n^2}$  converges.

#### 3. (1 pt) Library/maCalcDB/setSeries6CompTests/benny\_ser3B.pg

Each of the following statements is an attempt to show that a given series is convergent or divergent not using the Comparison Test (NOT the Limit Comparison Test.) For each statement, enter C (for "correct") if the argument is valid, or enter I (for "incorrect") if any part of the argument is flawed. (Note: if the conclusion is true but the argument that led to it was wrong, you must enter I.)

1. For all  $n > 1$ ,  $\frac{1}{n \ln(n)} < \frac{2}{n}$ , and the series  $2 \sum \frac{1}{n}$  diverges, so by the Comparison Test, the series  $\sum \frac{1}{n \ln(n)}$  diverges.
2. For all  $n > 2$ ,  $\frac{n}{n^3-7} < \frac{2}{n^2}$ , and the series  $2 \sum \frac{1}{n^2}$  converges, so by the Comparison Test, the series  $\sum \frac{n}{n^3-7}$  converges.
3. For all  $n > 2$ ,  $\frac{1}{n^2-2} < \frac{1}{n^2}$ , and the series  $\sum \frac{1}{n^2}$  converges, so by the Comparison Test, the series  $\sum \frac{1}{n^2-2}$  converges.
4. For all  $n > 1$ ,  $\frac{n}{5-n^3} < \frac{1}{n^2}$ , and the series  $\sum \frac{1}{n^2}$  converges, so by the Comparison Test, the series  $\sum \frac{n}{5-n^3}$  converges.
5. For all  $n > 2$ ,  $\frac{\ln(n)}{n^2} > \frac{1}{n^2}$ , and the series  $\sum \frac{1}{n^2}$  converges, so by the Comparison Test, the series  $\sum \frac{\ln(n)}{n^2}$  converges.
6. For all  $n > 1$ ,  $\frac{\sin^2(n)}{n^2} < \frac{1}{n^2}$ , and the series  $\sum \frac{1}{n^2}$  converges, so by the Comparison Test, the series  $\sum \frac{\sin^2(n)}{n^2}$  converges.

#### 4. (1 pt) Library/maCalcDB/setSeries6CompTests/benny\_ser4.pg

The three series  $\sum A_n$ ,  $\sum B_n$ , and  $\sum C_n$  have terms

$$A_n = \frac{1}{n^{10}}, \quad B_n = \frac{1}{n^4}, \quad C_n = \frac{1}{n}.$$

Use the Limit Comparison Test to compare the following series to any of the above series. For each of the series below, you must enter two letters. The first is the letter (A,B, or C) of the series above that it can be legally compared to with the Limit

Comparison Test. The second is C if the given series converges, or D if it diverges. So for instance, if you believe the series converges and can be compared with series C above, you would enter CC; or if you believe it diverges and can be compared with series A, you would enter AD.

$$\begin{aligned} \text{---1. } & \sum_{n=1}^{\infty} \frac{6n^2 + 6n^9}{4n^{10} + 8n^3 - 4} \\ \text{---2. } & \sum_{n=1}^{\infty} \frac{4n^4 + n^{10}}{935n^{14} + 8n^4 + 6} \\ \text{---3. } & \sum_{n=1}^{\infty} \frac{6n^6 + n^2 - 6n}{8n^{16} - 6n^{12} + 5} \end{aligned}$$

**5. (1 pt) Library/maCalcDB/setSeries6CompTests/ur\_sr.6.14.pg**

For each sequence  $a_n$  find a number  $k$  such that  $n^k a_n$  has a finite non-zero limit.

(This is of use, because by the limit comparison test the series  $\sum_{n=1}^{\infty} a_n$  and  $\sum_{n=1}^{\infty} n^{-k}$  both converge or both diverge.)

A.  $a_n = (2 + 2n)^{-5}$   
 $k =$  \_\_\_\_\_

B.  $a_n = \frac{5}{n^3 + n}$   
 $k =$  \_\_\_\_\_

C.  $a_n = \frac{2n^2 + 5n + 2}{5n^6 + 3n + 2}$   
 $k =$  \_\_\_\_\_

D.  $a_n = \left( \frac{2n^2 + 5n + 2}{5n^6 + 3n + 2\sqrt{n}} \right)^3$   
 $k =$  \_\_\_\_\_

**6. (1 pt) Library/maCalcDB/setSeries5IntegralTest/eva8.3.1.pg**

Compute the value of the following improper integral if it converges. If it diverges, enter INF if it diverges to infinity, MINF if it diverges to minus infinity, or DIV otherwise (hint: integrate by parts).

$$\int_1^{\infty} \frac{2 \ln(x)}{x^7} dx$$

Determine whether

$$\sum_{n=1}^{\infty} \left( \frac{2 \ln(n)}{n^7} \right)$$

is a convergent series. Enter C if the series is convergent, or D if it is divergent. \_\_\_\_

**7. (1 pt) Library/maCalcDB/setSeries5IntegralTest/ur\_sr.5.11.pg**

Test each of the following series for convergence by the Integral Test. If the Integral Test can be applied to the series, enter CONV if it converges or DIV if it diverges. If the integral test cannot be applied to the series, enter NA. (Note: this means that

even if you know a given series converges by some other test, but the Integral Test cannot be applied to it, then you must enter NA rather than CONV.)

$$\begin{aligned} \text{---1. } & \sum_{n=1}^{\infty} \frac{\ln(5n)}{n} \\ \text{---2. } & \sum_{n=1}^{\infty} \frac{6}{n \ln(5n)} \\ \text{---3. } & \sum_{n=1}^{\infty} n e^{9n} \\ \text{---4. } & \sum_{n=1}^{\infty} \frac{n+3}{(-9)^n} \\ \text{---5. } & \sum_{n=1}^{\infty} n e^{-9n} \end{aligned}$$

**8. (1 pt) Library/maCalcDB/setSeries3Convergent/ns8.3.17BB.pg**

Match each of the following with the correct statement. C stands for Convergent, D stands for Divergent.

$$\begin{aligned} \text{---1. } & \sum_{n=1}^{\infty} \frac{\ln(n)}{2n} \\ \text{---2. } & \sum_{n=1}^{\infty} \frac{1}{3 + \sqrt[7]{n^6}} \\ \text{---3. } & \sum_{n=1}^{\infty} \frac{6}{n(n+6)} \\ \text{---4. } & \sum_{n=1}^{\infty} \frac{4 + 3^n}{8 + 8^n} \\ \text{---5. } & \sum_{n=1}^{\infty} \frac{6}{n^2 - 49} \end{aligned}$$

**9. (1 pt) Library/maCalcDB/setSeries3Convergent/ns8.3.6.pg**

Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \left( \frac{1}{\sqrt[5]{n^3}} + \frac{4}{n^5} \right)$$

If convergent, enter the 6 th partial sum to estimate the sum of the series; otherwise, enter DIV.

(Note: if you have trouble reading this problem, try selecting typeset mode below and then hitting the submit answer button.)

**10. (1 pt) Library/maCalcDB/setSeries3Convergent/ns8.3.7.pg**

Match each of the following with the correct statement. C stands for Convergent, D stands for Divergent.

$$\begin{aligned} \text{---1. } & \sum_{n=2}^{\infty} \frac{8}{8n \ln(n)} \\ \text{---2. } & \sum_{n=1}^{\infty} \frac{n^6}{n^4 + 5} \end{aligned}$$

—3.  $\sum_{n=1}^{\infty} \frac{6}{n^4 + n^8}$

—4.  $\sum_{n=1}^{\infty} \frac{6 + 1^n}{2^n}$

—5.  $\sum_{n=1}^{\infty} ne^{-n^2}$