MAT 137-7 Exam 1 Name ______ 02/13/09 - Buena suerte! 60 points Calculators may be used, but you must show all work - unsupported answers (e.g., calculator output) will receive minimal credit

1.(8) Let R be the region bounded by $y = 3 - x^2$ and y = 2x. (Note: R extends into the first, second, and third quadrants.) Set up, but *do not evaluate*, integral expressions for the area of R, using

(a) vertical slices, integrating with respect to x.

(b) horizontal slices, integrating with respect to y. (Hint: Draw a large picture.)

2.(6) Set up, but do not evaluate, a definite integral for the volume of the solid pictured below. The base of the solid is the region bounded by $y = x^2$ and y = 4, and vertical cross-sections perpendicular to the y-axis are squares with one side in the base.



3.(10) Set up, but do not evaluate an integral for the volume of the solid obtained by rotating the region bounded by y = x and $y = 4x - x^2$ about

(a) the x-axis.



(b) the *y*-axis.

(c) the vertical line x = 3.

4.(8) Find the partial fractions decomposition of the rational function $f(x) = \frac{3x}{x^2 + x - 2}$.

5.(8) Consider a cylinder of height 10 and radius 3. Set up, but *do not evaluate*, an integral for the mass of the cylinder, if

(a) the density at a point is equal to $z^2 + 1$, where z is the distance from the point to the (circular) base of the cylinder.

(b) the density at a point is equal to $r^2 + 1$, where r is the distance from the point to the axis of the cylinder.

6.(10) Evaluate these indefinite integrals. Justify all steps in order to receive credit.

(a)
$$\int \frac{1}{(x^2+4)^{\frac{3}{2}}} dx$$
 (Set $x = 2\tan(u)$.)

(b)
$$\int \ln(x^2 + 1) dx$$
 (Hint: To get started, integrate by parts, with $dv = dx$.)

7.(6) (a) Find
$$\int x e^x dx$$
.

(b) Derive the reduction formula

$$\int x^n e^{ax} \, dx = \frac{1}{a} x^n e^{ax} - \frac{n}{a} \int x^{n-1} e^{ax} \, dx,$$

for n a positive integer and a a fixed real number.

8.(4) Show that $\int_{a}^{b} \ln(x) dx$ is equal to $\int_{\ln(a)}^{\ln(b)} ue^{u} du$, by changing variables ("pulling back") via the substitution $u = \ln(x)$. Here $0 < a < b < \infty$.