MAT 411-002 **Problem Set #1** Name _____

8/28/2019 (due Wednesday, 9/4/2019)

20 points

Please initial: I affirm that all work submitted in this assignment is my own original work and all work from other sources is acknowledged and cited.

1. Exercise 1.2.4 from the text.

2. Exercise 1.2.6 from the text.

   (a)

   (b)

3. Exercise 1.4.2 from the text.

4. Exercise 1.4.10 from the text.

5. Let G be the set of symmetries of the isosceles right triangle tiling of the plane, and let $r, s, t \in G$ be defined by $r(x, y) = (y, x)$, $s(x, y) = (x + 1, y)$, and $t(x, y) = (x, y + 1)$, as presented in class. We showed that any composite of elements of G is an element of G, and we showed that any composite g of r, s, and t, of any length, can be expressed as $g = s^a t^b r^c$, where $(a, b) \in \mathbb{Z} \times \mathbb{Z}$ and $c = 0$ or 1, with (a, b, c) uniquely determined.[1] The expression on the right is called the *normal form* of the element g.

(a) Show that $rs \neq sr$.

(b) Find the normal form of each of the following elements [2]:

(i) rsr^{-1} and rtr^{-1}.

(ii) srs^{-1} and trt^{-1}.

(iii) $rsr^{-1}s^{-1}$ and $rtr^{-1}t^{-1}$.

(iv) $rs^m t^n$, for $(m, n) \in \mathbb{Z} \times \mathbb{Z}$.

[1] Juxtaposed functions are meant to be composed, just as in class.

[2] gxg^{-1} is called the *conjugate* of x by g, often written x^g. $xyx^{-1}y^{-1}$ is called the *commutator* of x and y, often denoted $[x, y]$.