

# Throughput Optimization in Relay Networks Using Markovian Game Theory

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**Abstract**—In this paper, problem of throughput optimization in relay networks where all users transmit their packets on a multiple-access channel is studied by introducing a new Markovian game theoretical solution. Despite the previously reported works, in the proposed model, simultaneously transmitted packets on a multiple-access channel are not always discarded. In this article, the possibility of capturing one of these packets is considered. Both cases of cooperative and non-cooperative stochastic game solutions are investigated and compared. The main objective is to maximize the system throughput with minimum transmission delay and power consumption cost. Effect of different packet error rates due to possible collision occurrence is considered in the game definition that improves system performance further. Performance of the proposed non-cooperative game model approaches the cooperative case, however the non-cooperative game model adds less signaling load to the system, therefore it is more likely to be used in practical applications.<sup>1</sup>

Index Terms- Relay networks, packet forwarding, multiple-access channels, stochastic stationary games.

## I. INTRODUCTION

In wireless relay networks, when several transmitters randomly attempt to access a finite number of resources, simultaneous packet transmission is unavoidable. This in turn will produce interference and degrade system performance [1], [2]. Optimizing system performance in terms of high throughput and low latency is one of the challenging issues in wireless networks, where multiple access is essential.

Recently, game theoretical approaches are used to address this optimization problem, due to selfish behavior of the users as well as distributed nature of wireless cellular and ad-hoc networks. Another important advantage of this methodology is considering implementation cost in optimization process [3]. Game theory is a proper method to model packet forwarding in wireless networks and to analyze the trade off between users' interests to avoid forwarding each others packets due to limited power versus providing relay service in order to increase system throughput [4].

To study this issue, a three-node relay network consisting of a source node, a relay node and a destination node is considered in [5], [6]. Both source and relay nodes independently generate packets. Stochastic game theory is used to model packet forwarding in this system [5], [6]. In these works, simultaneous packet transmission by source and relay nodes results in collision and both nodes' packets will be discarded.

Moreover, in [5], only a single transmit buffer is assigned to the relay node in order to keep all different types of packets. Therefore, the relay node rejects the received packets from the source node whenever there is a packet in its transmit buffer. Consequently, source node's packet will be blocked and the channel remains unused, although there is no collision. This results in lower system throughput performance, which is not desirable in practical applications.

In this article, a similar case of a three-node relay network is considered. A new model with two different buffers for the relay node is introduced to allow the relay to keep its own packets and received packets from the source node separately. This solution provides the relay node with the option of accepting packets from the source node, even if it has a packet in its transmit queue. Another advantage of using two separate buffers in the proposed method is the possibility of taking a more flexible strategy by relay node. In this model, the relay node can decide to transmit either its own packet or the received packet from the source node, when both are in the queue. As a result, the relay node selects a proper strategy to maximize its utility considering the game conditions. This approach may provide a better performance, specially when the relay node has a high packet generation rate.

In addition, a new stochastic game theoretical model is proposed in this article that covers a more general case of multiple-access channels, where both simultaneously transmitted packets are not discarded. Interference is unavoidable in most applications, but it does not necessarily cause packet loss. The packet loss depends on several parameters, specially on the level of interference. For instance, in CDMA systems, the receiver captures the desired packet from a number of simultaneously interfering packets using orthogonality of codes. Even, in a multiple-access slotted ALOHA system, not any collision results in a packet loss [7]. Considering packet loss rate in the proposed game definition shows considerable increase in system performance in terms of throughput and latency.

The Markovian game model is investigated for both cases of cooperative and non-cooperative game solutions. In non-cooperative game solution, both nodes behave selfishly and try to maximize their own utility and optimize their individual performance in terms of throughput maximization and delay minimization. This is to obtain the best response Nash equilibrium strategy. In cooperative game, the goal for both nodes is to optimize the total system performance. Therefore, both nodes cooperate with each others to maximize the total system utility and obtain the social equilibrium strategy set.

The rest of this paper is organized as follows. In section

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II, stochastic game theory is studied. System model for the proposed relay network is presented in section III. In section IV, the proposed game definitions consisting of available action sets of each node, stationary transition matrix and utility functions of players are presented. Numerical results are provided in section V, followed by conclusions in section VI.

## II. STOCHASTIC GAMES

In this section, class of stochastic games, which is the basis of our proposed analysis model is studied. In stochastic games, complete history of the game in each round is summarized in a state that follows a Markov process. A Markovian game can be modeled with a  $n \times n$  transition matrix, denoted by  $T$ , where  $n$  represents the number of states. Each element of this matrix,  $p_{ij}$  shows the probability of moving from state  $i$  in time  $n$  to state  $j$  in time  $n + 1$ , where  $p_{ij} \geq 0, \forall i, j$ , and  $\sum_{j=1}^n p_{ij} = 1, \forall i$ .

A discrete time stochastic game with  $N$  players is denoted by  $(Q, \{A_i\}_{i=1}^N, \{U_i\}_{i=1}^N, t)$  [8], where

- $Q$ , is the Borel state space.
- $A_i$ , is action set of player  $i$ , and  $A = A_1 \times \dots \times A_N$ , denotes the action profile of all players.
- $U_i : Q \times A \rightarrow \mathfrak{R}$ , where  $\mathfrak{R}$  is Real set.  $U_i$  determines the immediate utility function of player  $i$ , which depends on the current state and the action profile of the game.
- $t : Q \times A \rightarrow [0, 1]$ , is the transition probability function.

Mixed strategy of player  $i$  is denoted as  $s_i$ . Strategy profile of the game in the  $n^{th}$  time slot is denoted as  $S = (S_1, \dots, S_N)$ , where  $S_i$  is the strategy set of player  $i$ .

Solution of the game achieves the Nash equilibrium strategy set of all players when each rational player selects its best possible response to other players' strategies, provided that neither player can increase its utility by unilaterally changing its strategy. A strategy profile,  $S^*$  achieves Nash equilibrium iff [9],

$$\forall i \in N, \forall s_i \in S_i, U_i(s_i^*, s_{-i}^*) \geq U_i(s_i, s_{-i}^*) \quad (1)$$

where  $s_{-i}$ , denotes the strategy of all players except player  $i$ .

A strategy is called stationary if the strategy profile of the  $n^{th}$  time slot shown by  $(S^n)$  only depends on the current state of the game rather than on the complete game history. The stationary strategy sets of all players is denoted by  $\delta = (\delta_1, \dots, \delta_N)$ . Notation  $\Pi(\delta)$  presents the stationary probability distribution over states, such that  $\Pi(\delta) = \Pi(\delta) \times T(\delta)$ , where  $T(\delta)$  is the state transition matrix and  $\times$  is defined as matrix multiplication.

In stationary stochastic games, the expected utility function of player  $i$  is driven as follows [5].

$$U_i(\delta) = \sum_{q_k \in Q} \Pi_k(\delta) E[U_i(q_k, \delta)] \quad (2)$$

where,  $E[U_i(q_k, \delta)]$  is the expected utility of player  $i$  in the  $k^{th}$  state over the stationary strategy  $\delta$ .

A stochastic game with finite number of states and actions has a Nash equilibrium [9]. In this paper, a two-player

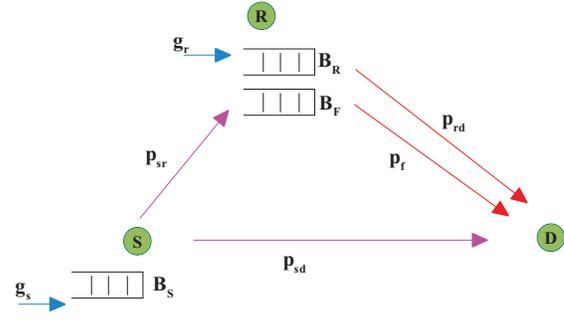


Fig. 1. System model for the proposed relay network, where  $S, R$  and  $D$  represent source, relay and destination nodes, respectively.

Markov game with finite number of states and finite number of possible actions for each player is defined to model the packet forwarding in a basic relay network. Both cases of cooperative and non-cooperative games are considered. In non-cooperative game, each player  $i$  selfishly maximizes its own utility function,  $U_i(\delta)$  to reach the best response Nash equilibrium strategy, based on equation (2), while in cooperative games, players collaborate with each other to jointly maximize the total utility of the game.

## III. SYSTEM MODEL

In this article, a two-hop relay network consisting of a source node, a relay node and a destination node is considered as depicted in fig. 1. It is assumed that this relay node is the best intermediate node of the network for the source node considering energy efficiency, path length and link quality. The study of algorithms to find the best relay node from available intermediate nodes is out of the scope of this article. It is assumed that the best relay node among all available relay nodes in network has been selected.

Source and relay nodes randomly access the common channel to communicate with destination where simultaneous transmission may result in a collision. It is assumed that both source and relay nodes broadcast the number of packets in their buffers at the end of each time slot. The broadcast information includes the delivery status of the packets as well as the channel quality. This assumption enables transmitters to adjust their transmit power ensuring successful packet delivery to the destination when no collision occurs. Therefore, packets leave the transmit buffer after successful transmission with appropriate power level. In the case of packet transmission failure in any time slot, packets remain in the buffer until they are transmitted in the next time slots by an Automatic Repeat-Request (ARQ) retransmission protocol.

The system is modeled as a two-player Markovian game, denoted by  $S$  and  $R$ , representing source and relay nodes, respectively. Source node has a single transmit buffer which is called *source buffer* hereafter to keep the generated packets prior to sending. Relay node has two transmit buffers. The first buffer of relay node is called *internal buffer* that contains generated packets at the relay node. The second one is called *forward buffer*, which contains received packets from source node in the previous time slot. Packets are generated at source and relay nodes independently by rates  $g_s$  and  $g_r$ , respectively. Packets are generated provided that nodes' buffers are empty at the end of previous time slot [5]. The occupancy status

of source buffer, internal and forward buffer of relay node presented by  $\{B_S, B_R, B_F\}$  defines the state of the game.

The proposed game is modeled as a complete information game, where each player knows the current state and available action sets as well as the outcomes of actions for all the players. Considering the information provided by the broadcast channel, both players can observe the current state of the game in each time slot that is defined by the number of packets in the buffers. Since the number of packets in the nodes' buffers only include eight different states, hence broadcasting this parameter does not impose a considerable signaling overload to the system. However, this assumption makes the proposed solution applicable in decentralized networks. Since there is no need to have a central station to control the network and each node can independently select its best strategy set considering the game conditions. In non-cooperative game, players are not aware of the selected action by other nodes. For instance, they do not know whether their opponent attempts to send its packet or not, therefore in each time slot, they select their actions based on the game status, including current state, packet transmit energy, packet arrival rate, as well as cooperation reward regardless of the action actually taken by the other node.

#### A. Collision robust channel model

In most developed models for relay networks, both source and relay nodes' packets are dropped in case of simultaneous transmission due to collision. In this section, a new model is introduced to address a more general case in which one of the simultaneously transmitted packets can be captured based on physical layer parameters.

In interference channels, signal to interference and noise ratio (SINR) denoted by  $\gamma$ , is defined as [7]

$$\gamma = \frac{h_i \cdot P_i}{N_0 + \frac{1}{L} \sum_{j=1, j \neq i}^N h_j \cdot P_j} \quad (3)$$

where  $P_i$  is the transmission power of node  $i$ ,  $h_i$  is the channel gain between transmitter  $i$  and the receiver,  $N_0$  is the noise power at the receiver and  $L$  is the processing gain, where  $L = 1$  is used for narrowband and  $L \gg 1$  for wide band systems. In the sequel,  $i = 1$  is assigned to source node, and  $i = 2$  is considered for relay node, unless explicitly specified otherwise.

Probability of packet error is a function of  $\gamma$ , which means  $PER = f(\gamma)$ , where  $f$  is a system dependent function depending on transmission parameters such as transmitter structure, receiver sensitivity level, modulation scheme, channel coding, data transmission rate and packet length [10].

SINR should be higher than a system dependent threshold level,  $\Gamma$ , to guarantee error free delivery of packets to the destination [7]. Both source and relay nodes adjust their powers such that the SINR level at destination is higher than  $\Gamma$ , provided that there is no interfering node. Hence, minimum transmission power,  $P_i^t(min)$  of the  $i^{th}$  node is determined as follows,

$$\begin{aligned} \gamma_i &= \frac{P_i^r}{N_0} = \frac{P_i^t \cdot h_i}{N_0} \geq \Gamma \\ &\Rightarrow P_i^t(min) = \Gamma \cdot N_0 / h_i \end{aligned} \quad (4)$$

where  $P^t$  and  $P^r$  denote the transmission and reception power levels, respectively.

If both nodes attempt to transmit simultaneously with the minimum power, the  $\gamma$  value of node  $i$  is converted to,

$$\begin{aligned} \gamma_i &= \frac{P_i^r}{N_0 + P_i^r} = \frac{P_i^t \cdot h_i}{N_0 + P_i^t \cdot h_i} \\ &= \frac{\Gamma}{\Gamma + 1}, \quad \bar{i}, i = 1, 2, \bar{i} \neq i. \end{aligned} \quad (5)$$

This degradation at SINR level may severely affect signal reception depending on transmission technology. This effect in some cases such as uncoded BPSK is more visible and almost all the packets are lost, while in systems such as DS-CDMA performance of system does not change considerably [13].

The following joint probabilities are defined to model packet error rate in case of collision for arbitrary transmission techniques:

- $p^R(s, \bar{r}; d)$ , is the probability of successful reception of only source node packet when collision occurs at destination.
- $p^R(\bar{s}, r; d)$ , is the probability of successful reception of only relay node packet when collision occurs at destination.
- $p^R(s, r; d)$ , is the probability of successful reception of both packets.
- $p^R(\bar{s}, \bar{r}; d)$ , is the probability of failure of both packets.

Consequently, the probability of failure in delivery of source and relay nodes' packets are calculated as,

$$p^E(s; d) = p^R(\bar{s}, r; d) + p^R(\bar{s}, \bar{r}; d) \quad (6)$$

$$p^E(r; d) = p^R(s, \bar{r}; d) + p^R(\bar{s}, \bar{r}; d) \quad (7)$$

These joint probabilities are used to find the state transition matrix and also expected utility functions of the players for the proposed game model.

#### B. Average System throughput

Without loss of generality and to simplify system throughput calculations, it is assumed that nodes randomly access the channel using slotted ALOHA access protocol. Other protocols such as ALOHA and reservation ALOHA can also be applied to this system. The probability of successful packet transmission by node  $i$  at state  $q$ ,  $\mu_i(q)$  is calculated as

$$\mu_i(q) = p_i^S(q) \cdot [(1 - p_i^S(q)) + p_i^S(q) \cdot p^R(i, \bar{i}; d)] \quad (8)$$

where  $p_i^S(q)$  denotes the probability of sending a packet by node  $i$  at state  $q$ .

The average throughput of each node  $i$ , denoted by  $\bar{\mu}_i$  is calculated as follows,

$$\bar{\mu}_i = E\left(\sum_{q=1}^8 \Pi_q \times \mu_i(q)\right) \quad (9)$$

where  $\Pi_q$  is the probability of state  $q$  of the game. The average throughput of the system,  $\bar{\mu}$  is summation of the source and relay nodes' throughputs, ( $\bar{\mu} = \sum_{i=1}^2 \bar{\mu}_i$ ).

### C. Average transmission delay

One other important specification of networks is the average delay of packet transmission. In some applications, a certain level of transmission delay is acceptable. If a packet remains at each transmit buffer, a delay counter is set for the corresponding node. The average packet transmission delay of both source and relay nodes presented by  $\bar{d}_i$  as well as the average delay of system denoted by  $\bar{d}$  are defined as,

$$\bar{d}_i = E\left(\sum_{q=1}^8 \Pi_q \times d_i(q)\right) \quad (10)$$

$$\bar{d} = \frac{\sum_{i=1}^2 g_i \cdot \bar{d}_i}{\sum_{i=1}^2 g_i} \quad (11)$$

where  $g_i$  is the packet generation rate at node  $i$  and  $d_i(q)$  is the probability of keeping a packet at transmit buffer of node  $i$  at state  $q$ .

## IV. MARKOVIAN GAME MODEL

The proposed stationary stochastic game model is described in this section, providing the strategy set of players, as well as transition probability matrix and the expected utility function of the players.

### A. Strategy set of players

In this section, stationary strategy sets of players are described. The mixed stationary strategies of players is defined as the probability distribution of the available actions. Strategy space of source node is denoted by  $(p_{sd}, p_{sr}, p_{sw})$ , where

- $p_{sd}$ , is probability of sending a packet from the source node to the destination node.
- $p_{sr}$ , is probability of sending a packet from the source node to the relay node.
- $p_{sw}$ , is probability of waiting at the source node.

Since the summation of these probabilities is one, two of them completely define strategy set of the source node.

Strategy set of relay node is denoted by  $(p_{rd}, p_f, p_{rw}, p_{ac}, p_r)$ , which consists of probabilities of different actions taken by relay node including

- $p_{rd}$ , is probability of sending packet from the relay node to the destination.
- $p_f$ , is probability of forwarding the received packet to the destination.
- $p_{rw}$ , is probability of waiting at the relay node.
- $p_{ac}$ , is probability of accepting the received packet from the source node.
- $p_r$ , is probability of rejecting the received packet from the source node.

Relay node does not accept a packet from source node, if there is a packet in the forward buffer. It can either transmit its own packet or transmit the received packet (if there is any) or wait, when the forward buffer is empty.

Since,  $p_{rd} + p_f + p_{rw} = 1$ , and  $p_{ac} + p_r = 1$ , therefore,  $(p_{rd}, p_f, p_{ac})$  can sufficiently capture the strategy set of relay node. Consequently, the probability vector  $(p_{sd}, p_{sr}, p_{rd}, p_f, p_{ac})$  models the strategy profile of the game.

The mixed strategy profile of the game regarding the current state of the game is summarized in Table I.

TABLE I  
STRATEGY SET OF PLAYERS

Current State of the Game ( $B_S, B_R, B_F$ )	Strategy Set of Players ( $p_{sd}, p_{sr}, p_{rd}, p_f, p_{ac}$ )
$s_1 = (0, 0, 0)$	$(0, 0, 0, 0, p_{ac})$
$s_2 = (0, 0, 1)$	$(0, 0, 0, p_f, 0)$
$s_3 = (0, 1, 0)$	$(0, 0, p_{rd}, 0, p_{ac})$
$s_4 = (0, 1, 1)$	$(0, 0, p_{rd}, p_f, 0)$
$s_5 = (1, 0, 0)$	$(p_{sd}, p_{sr}, 0, 0, p_{ac})$
$s_6 = (1, 0, 1)$	$(p_{sd}, p_{sr}, 0, p_f, 0)$
$s_7 = (1, 1, 0)$	$(p_{sd}, p_{sr}, p_{rd}, 0, p_{ac})$
$s_8 = (1, 1, 1)$	$(p_{sd}, p_{sr}, p_{rd}, p_f, 0)$

### B. Transition probability matrix

Transition probability from state  $i$  to  $j$  in consecutive time slots for stationary strategy profile ( $\delta$ ) is denoted as  $T_{ij}(\delta), i, j \in \{1, \dots, 8\}$ .

For instance, the fifth row of the transition probability matrix is presented here as detailed in [6].

$$\begin{aligned} T_{51}(\delta) &= (1 - g_s)(1 - g_r) \cdot p_{sd}, \\ T_{52}(\delta) &= (1 - g_s)(1 - g_r) \cdot p_{sr} \cdot p_{ac}, \\ T_{53}(\delta) &= g_r(1 - g_s) \cdot p_{sd}, \quad T_{54} = g_r(1 - g_s) \cdot p_{sr} \cdot p_{ac}, \\ T_{55}(\delta) &= (1 - g_r) \times [1 - p_{sr} - p_{sd} + g_s \cdot p_{sd} + p_{sr}(1 - p_{ac})], \\ T_{56}(\delta) &= g_s(1 - g_r) \cdot p_{sr} \cdot p_{ac}, \\ T_{57}(\delta) &= g_r \times [1 - p_{sr} - p_{sd} + g_s \cdot p_{sd} + p_{sr}(1 - p_{ac})] \\ T_{58}(\delta) &= g_s \cdot g_r \cdot p_{sr} \cdot p_{ac}. \end{aligned} \quad (12)$$

### C. Expected payoff functions

Player's payoff function is defined as the difference between obtained rewards and paid costs for a specific strategy set. According to (2), utility function of each player in stationary Markovian games is defined as the expected value of utility over the state distribution.

The assigned costs and rewards of the proposed game model are defined as follows:

- $R^d$ , is delivery reward for successful delivery of a packet to the destination node.
- $R^f$ , is forwarding reward paid by the source node for accepting the received packet by relay node.
- $C_{ij}^t$ , is transmission cost, for transmission of a single packet from node  $i$  to node  $j$ .
- $C^k$ , is keeping cost for keeping the packet in the buffer (waiting), and also retransmission of the corrupted packets due to collision.
- $C^{kf}$ , is keeping forward cost paid by the relay node for avoidance of received packet delivery.

- $C^r$ , is relaying delay cost paid by the source node for latency imposed to the system due to transmitting packets via the relay node.

A selfish relay node intends to transmit its own packets and avoid cooperation by rejecting the source node packets. Forwarding reward is utilized to encourage relay node to accept source node's packets. However, source node receives the delivery reward,  $R^d$  when the relay node successfully transmits the source node packet to the destination.

In this game definition, keeping forward cost is selected to be greater than keep cost  $C^k$ , in order to encourage the relay node to transmit packets in forward buffer with higher priority than its own packets. Otherwise, the relay node performs in a selfish manner and transmits its own packets.

According to aforementioned definitions, utility functions of both source and relay nodes are calculated as detailed in [6]. Utility function of source node is presented in the appendix.

## V. NUMERICAL RESULTS

Numerical results are provided in this section to investigate system performance for both cooperative and non-cooperative schemes in terms of achieved utility, average throughput and average transmission delay for different parameters. In the simulations the game parameters are defined as  $C_{sd}^t = 0.4$ ,  $C_{sr}^t = C_{rd}^t = 0.1$ ,  $g_s = g_r = 0.25$ ,  $R^d = 1$ ,  $R^f = 0.4$ ,  $C^k = 0.1$ ,  $C^{kf} = 0.2$ ,  $C^r = 0.05$ .

TABLE II  
STRATEGY PROFILE OF THE GAME ANALYSIS FOR DIFFERENT TRANSMISSION COSTS FROM SOURCE TO DESTINATION NODE

Transmission cost from source to destination, $C_{sd}^t = 0.3$	Transmission cost from source to destination, $C_{sd}^t = 0.5$
$p_{sd} = 0.65$	$p_{sd} = 0.35$
$p_{sr} = 0.15$	$p_{sr} = 0.35$
$p_{rd} = 0.7$	$p_{rd} = 0.35$
$p_f = 0.1$	$p_f = 0.45$
$p_{ac} = 0.85$	$p_{ac} = 0.65$

Different strategy sets taken by source and relay nodes in the non-cooperative game versus different transmission cost between source and destination,  $C_{sd}^t$  are presented in Table II. The tendency of source node to transmit its packet via the relay node rather than directly communicating with

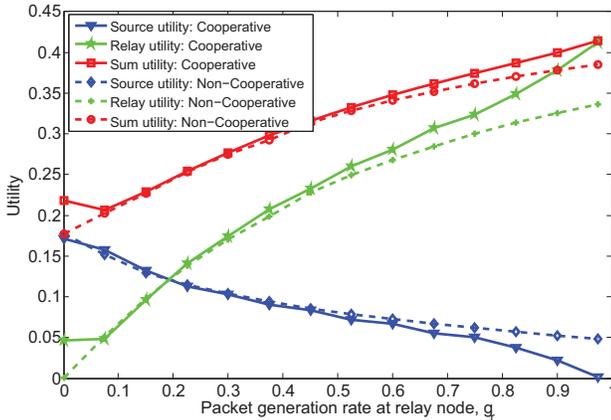


Fig. 2. Utility of source and relay nodes versus packet generation rate at relay node for cooperative and non-cooperative games.

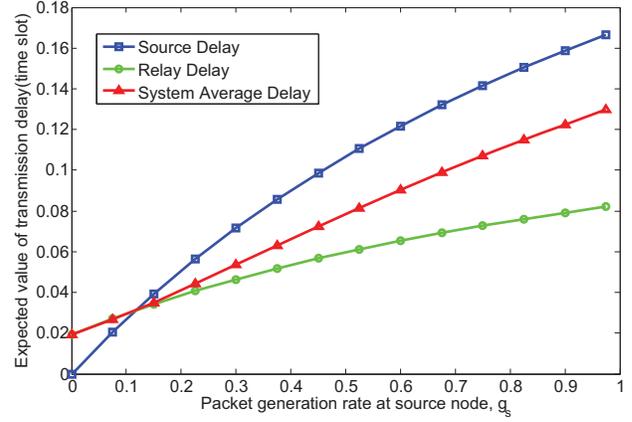


Fig. 3. Average packet transmission delay of system versus packet generation rate at source node.

destination increases as the cost of direct communication increases. Sending packets via the relay node although may increase the transmission delay of packets, maximizes the overall utility due to lower transmission energy cost.

Fig. 2, shows the utility of both source and relay nodes as well as summation of their utilities versus packet generation rate of the relay node. Increasing packet generation rate in relay node increases its utility, since it gets more packet delivery reward. On the other hand, since the relay node attempts to occupy the channel more often, it causes collision with source node packet transmission. Relay node's incentive to cooperate with the source node is reduced because of trade-off between sending its own packets and received packets from the source node that results in reduction in source node utility. However, the summation utility increases for higher packet generation rate in relay node since the increase in the relay node's utility is dominant. A similar result is obtained for packet generation rate at the source node.

In cooperative game, nodes jointly select the strategy profile in order to maximize the total utility. While, in non-cooperative, nodes try to maximize their own payoffs. Therefore, the summation of nodes' utilities in cooperative game is usually greater than non-cooperative game. Fig. 2, demonstrates that performance of the proposed non-cooperative scheme approaches cooperative system performance.

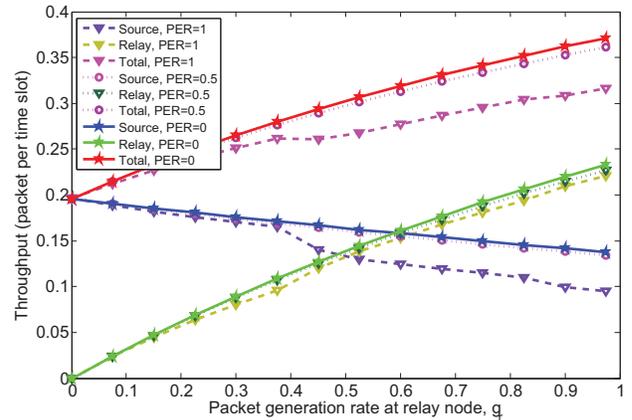


Fig. 4. Average throughput of system versus packet generation rate at relay node for different packet error rates.

Average delay of the proposed system is depicted in fig. 3. The average delay of system is increased for higher packet generation rates at source and relay nodes as expected. When packet generation rate is increased, players intend to keep their packets rather than transmitting them to the destination immediately to avoid collision. This results in less energy consumption for retransmission. For higher packet generation rates, it is more likely for a packet to be retransmitted due to possibility of collision which is another cause of delay.

Fig. 4 compares the average throughput of the proposed system for different probabilities of packet discard in case of collision, which is shown by  $PER$  in this figure. The result demonstrates that the maximum achievable throughput is higher for the systems that are robust to channel interference and the probabilities of packet discarding in the case of collision is lower. The proposed system is flexible in the sense that players take more appropriate strategies considering packet loss probability.

## VI. CONCLUSIONS

In this article, a stochastic stationary game theoretical model is introduced in order to optimize throughput of a relay network. The strategy profile shows system behavior in different conditions while utility of system is obtained to study the overall system performance in terms of higher throughput, lower latency and minimum energy cost. Game theoretical modeling provides us with the possibility of optimizing the system performance with respect to multiple criteria.

Employing two separate buffers at a relay node enables the players to select more appropriate strategies in order to optimize the overall system throughput under different states of the system. In contrast to traditional models where the packets are assumed to be lost in the case of collision, the proposed model is more flexible in the sense that the possibility of capturing one of the simultaneously transmitted packets is considered. Therefore, the proposed scheme is applicable to a wide variety of relay networks with different transmission techniques and packet error rates. The system throughput is improved further by increasing the buffers' lengths at price of higher system complexity.

Simulation results demonstrate that performance of the proposed non-cooperative system approaches the cooperative system. However, in non-cooperative game model, the players do not need to know each other's strategies, rendering this method to be suitable for most practical applications of wireless networks.

### Appendix: Expected payoff function of source node

In this appendix, the utility function of source node is calculated, considering the defined costs and rewards for the proposed game model, the joint probabilities of the successful packet reception and the probabilities of failure in packet delivery.

$$\begin{aligned}
U_1(\delta) = & \Pi_2(\delta) \times \{ p_f \cdot R^d \} + \Pi_4(\delta) \times \{ p_f \cdot R^d \} \\
& + \Pi_5(\delta) \times \{ p_{sd}(R^d - C_{sd}^t) + p_{sr} \cdot p_{ac}(-R^f - C_{sr}^t - C^r) \\
& + p_{sr}(1 - p_{ac})(-C^k - C_{sr}^t) + (1 - p_{sr} - p_{sd})(-C^k) \} \\
& + \Pi_6(\delta) \times \{ p_{sd}(1 - p_f + p_f \cdot p^R(s, \bar{r}; d))(R^d - C_{sd}^t) \\
& + p_{sd} \cdot p_f \cdot p^R(\bar{s}, \bar{r}; d)(-C^k - C_{sd}^t) \\
& + p_{sd} \cdot p_f \cdot p^R(\bar{s}, r; d)(R^d - C_{sd}^t - C^k) \\
& + p_{sr}(-C^k - C_{sr}^t) + p_{sr} \cdot p_f(1 - p^E(r; d))(R^d) \\
& + (1 - p_{sr} - p_{sd}) \times [p_f(R^d - C^k) + (1 - p_f)(-C^k)] \} \\
& + \Pi_7(\delta) \times \{ p_{sd}(1 - p_{rd} + p_{rd} \cdot p^R(s, \bar{r}; d))(R^d - C_{sd}^t) \\
& + p_{sd} \cdot p_{rd} \cdot p^R(\bar{s}, r; d)(-C^k - C_{sd}^t) \\
& + p_{sr}(1 - p_{rd}) \cdot p_{ac}(-R^f - C_{sr}^t - C^r) \\
& + p_{sr}(1 - p_{ac} + p_{rd} \cdot p_{ac})(-C^k - C_{sr}^t) \\
& + (1 - p_{sr} - p_{sd})(-C^k) \} \\
& + \Pi_8(\delta) \times \{ p_{sd}(1 - p_{rd} - p_f + (p_{rd} + p_f) \cdot p^R(s, \bar{r}; d)) \cdot \\
& (R^d - C_{sd}^t) + p_{sd}(p_{rd} + p_f)(1 - p^R(s, \bar{r}; d))(-C^k - C_{sd}^t) \\
& + p_{sd} \cdot p_f \cdot p^R(\bar{s}, r; d)R^d \\
& + p_{sr}(-C_{sr}^t - C^k) + p_{sr} \cdot p_f(1 - p^E(r; d))(R^d) \\
& + (1 - p_{sr} - p_{sd})(-C^k) + (1 - p_{sr} - p_{sd}) \cdot p_f(R^d) \} \quad (13)
\end{aligned}$$

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