

# Game Theoretic Study of Cooperative Spectrum Leasing in Cognitive Radio Networks

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## ABSTRACT

*In this paper, a novel property-right spectrum leasing solution based on Stackelberg game is proposed for Cognitive Radio Networks (CRN), where part of the secondary users present probabilistic dishonest behavior. In this model, the Primary User (PU) as the spectrum owner allows the Secondary User (SU) to access the shared spectrum for a fraction of time in exchange for providing cooperative relaying service by the SU. A reputation based mechanism is proposed that enables the PU to monitor the cooperative behavior of the SUs and restrict its search space at each time slot to the secondary users that do not present dishonest behavior in the proceeding time slots. The proposed reputation-based solution outperforms the classical Stackelberg games from both primary and reliable secondary users' perspectives. This novel method of filtering out unreliable users increases the PU's expected utility over consecutive time slots and also encourages the SUs to follow the game rule.*

*Keywords: Cognitive Radio Networks, Cooperative Communications, Primary User (PU), Spectrum Leasing, Stackelberg Game*

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## 1. INTRODUCTION

Due to the increasing number of users in contemporary communication systems, the demand for spectrum is growing very fast. However, the Federal Communications Commission's (FCC) technical report ((FCC), 2002) reveals that a considerable portion of spectrum remains unused over time. This suggests that the traditional fixed spectrum allocation techniques are not efficient. Thereby, the concept of cognitive networking is recognized as a promising solution

to provide the chance of access to the licensed spectrum by the unlicensed users, while the spectrum is not occupied by the Primary Users (PU) (Mitola, J., & Maguire, J.G.Q., 1999).

Two general approaches to cognitive radio networks are *common models* and *property-right models*. In common models, the Primary User (PU) is oblivious to the existence of the Secondary Users (SUs). The SUs monitor the licensed band to capture the holes (idle frequency bands) in the spectrum which are not utilized by the PUs. This method is sensitive to

the utilized spectrum sensing technique, since an untimely spectrum access by the secondary users may deteriorate the underlying interference management scheme and severely impact the PU operation. Therefore, this approach is not suitable for practical coexistence of networks.

On the other hand, in the property-right models, the PU willingly allocates some part of the licensed spectrum to the SUs in exchange for their relaying service Simeone, O., Gambini, J., Bar-Ness, Y., and Spagnolini, U. (2007). This technique brings about the efficient spectrum utilization which benefits both the primary and secondary users. The cooperative packet transmission enhances the PU's throughput, especially when there is no reliable direct link between the primary transmitter and its target receiver. In return, the SUs obtain the chance to access to a part of the spectrum.

A spectrum leasing scheme is proposed in Simeone, O., Stanojev, I., Savazzi, S., Bar-Ness, Y., Spagnolini, U., and Pickholtz, R. (2008), where a PU allocates the channel to the users of a secondary ad hoc network for a fraction of time, and the secondary network in return cooperates in forwarding the PU's packets using distributed space-time coding technique. A Stackelberg game model is used in this model, where the PU selects the fractions of time to be used for transmissions of the primary and network of the secondary users as well as the time for cooperative services, with the objective of maximizing its own transmission rate. In the next stage, the SUs that all transmit simultaneously compete with one another to set the optimal power allocation which results in a highest transmission rate.

A priced-based game model for spectrum leasing is proposed in Wang, X., Ma, K., Han, Q., Liu, Z., and Guan, X. (2012), where time allocation parameters as well as the price of spectrum are set by the PU, while the selected SU may increase its transmission rate by optimizing its transmission power. In Afghah, F., Costa, M., Razi, A., Abedi, A., and Ephremides, A. (2013), a cognitive radio network consisting of a single primary and a single secondary nodes is considered and a reputation-based Stackelberg game model is proposed, where

the primary and secondary jointly decide about the time allocation of the spectrum. This model accounts for energy efficiency and fairness to optimally split the time into three phases: i) PU transmission, ii) cooperative relaying, and iii) SU transmission.

In the previous reported work, it is assumed that the SUs are trustable in the sense that they use the same power for transmissions of their own packets as well as cooperative packet transmission for the PU (Simeone, O., Stanojev, I., Savazzi, S., Bar-Ness, Y., Spagnolini, U., & Pickholtz, R., 2008, Wang, X., Ma, K., Han, Q., Liu, Z., & Guan, X., 2012, Simeone, O., Gambini, J., Bar-Ness, Y., & Spagnolini, U., 2007, Hao, X., Cheung, M.H., Wong, V., Leung, V.C.M., 2011). However, this assumption may be violated in reality as cooperation is not an inherent characteristic of the cognitive users and they may prefer to save their limited available resources for their own packet transmission. In other words, although after granting the spectrum access, the SUs are supposed to treat the received packets from the primary similar to their own packets and forward them with an acceptable power, they may deviate from this rule and assign a low power to relay the PU's packet and reserve the remaining power for their individual transmission.

This overlooked issue of potential misbehavior of secondary users after accessing the channel is addressed in this paper. We consider a cognitive radio network with a single PU and  $N$  SUs. A cooperative credit is assigned to each SU that keeps up a record of its performance in cooperation with the PU. The cooperative credit is increased upon honest performance of the SU in relaying the PU's packet with an acceptable power. The cooperative credit of a SU is decreased if it allocates a lower power than expected to cooperative relaying. The proposed model provides the opportunity of recognizing the reliable SUs for the PU.

Another contribution of this paper is that although the proposed game is a one-shot Stackelberg game, however it has the characteristic of monitoring cooperative behavior of players over the time. Therefore, the proposed game

represents the important property of repeated games to enforce the players to obey the game rules and prevent selfish misbehavior, while it only saves the cooperative credit parameter of the SUs rather than keeping the whole history of the SUs' actions over all rounds of the game. Hence, the proposed reputation-based one-shot game is simpler and faster than repeated games and requires considerably less memory.

It is worth noting that the proposed model notably reduces the signaling overhead in selecting the secondary relays compared to previously reported work. In Simeone, O., Stanojev, I., Savazzi, S., Bar-Ness, Y., Spagnolini, U., and Pickholtz, R. (2008), at each time slot the secondary relays are selected from all available SUs noting their channel quality. This requires knowing the channel conditions for all secondary users and performing an exhaustive search over all  $2^N$  possible subsets of the SUs. This imposes a heavy signaling and computations and considerable latency to the systems that limits the scalability of this model. In our proposed model, at each time slot, the PU observes the cooperative credits of all SUs and selects  $K$  of them with highest cooperative credits. This considerably reduces the size of the search space from  $N$  to  $K$ , where  $K \ll N$ . This reduces signaling load compared to other work since only the channel conditions of the  $K$  selected secondary users need to be known by the primary.

The rest of this paper is organized as follows. In section 2, the system model for the proposed cognitive radio network is presented. In section 3, a brief overview on Stackelberg game is presented. The proposed Stackelberg game model for this scenario is described in section 4 followed by the equilibrium analysis in section 5. Numerical results and conclusions are provided in sections 6 and 7, respectively.

## 2. SYSTEM MODEL

In this paper, we propose a model for cooperative spectrum leasing, where multiple SUs co-exist with a single PU as depicted in Figure

1. The primary transmitter and receiver are denoted by  $PT$  and  $PR$ , respectively. Similarly,  $ST_i$  and  $SR_i$  represent the transmitter and receiver associated with secondary user  $i$ . In each time slot of the game, the PU selects a group of  $K$  reliable SUs among all  $N$  active SUs. This pool is presented by a blue circle in Figure 1. Then considering the quality of channels between the primary,  $PT$  and the secondary transmitters,

$$ST_i, i \in \{i_1, i_2, \dots, i_K\} \subset \{1, 2, \dots, N\},$$

the best SU is selected by the primary. The selected secondary node is denoted by  $S_k$  through this paper. The PU willingly allocates a portion of time slot to the selected secondary in exchange for relaying service.

The detailed of the proposed model is mentioned in section IV. In the proposed model, each time slot  $T$  is divided into the following three phases as depicted in Figure 2.

**Phase I:** Only the PU transmits its data for  $(1 - \alpha)T$  seconds,  $(0 \leq \alpha \leq 1)$ ;

**Phase II:** The selected SU relays the PU's data to  $PR$  for  $\alpha\beta T$  seconds,  $(0 \leq \beta \leq 1)$ ;

**Phase III:** The selected SU transmit its own data for  $\alpha(1 - \beta)T$  seconds.

For the sake of notation simplicity, we set  $T = 1$ .

Slow Rayleigh fading channels are assumed between the nodes, where the channel gains are invariant over one time slot.

The complex-valued channel coefficient are defined as follows

$h_p$ : Channel coefficient between  $PT$  and  $PR$

$h_{SP_i}$ : Channel coefficient between  $PT$  and

$ST_i$

$h_{SP}$ : Channel coefficient between  $ST_i$  and  $PR$

Figure 1. System model: coexistence of a single primary source-destination link and multiple secondary links. The blue circle encompasses the SUs with acceptable credit history that are candidates for cooperative relaying.

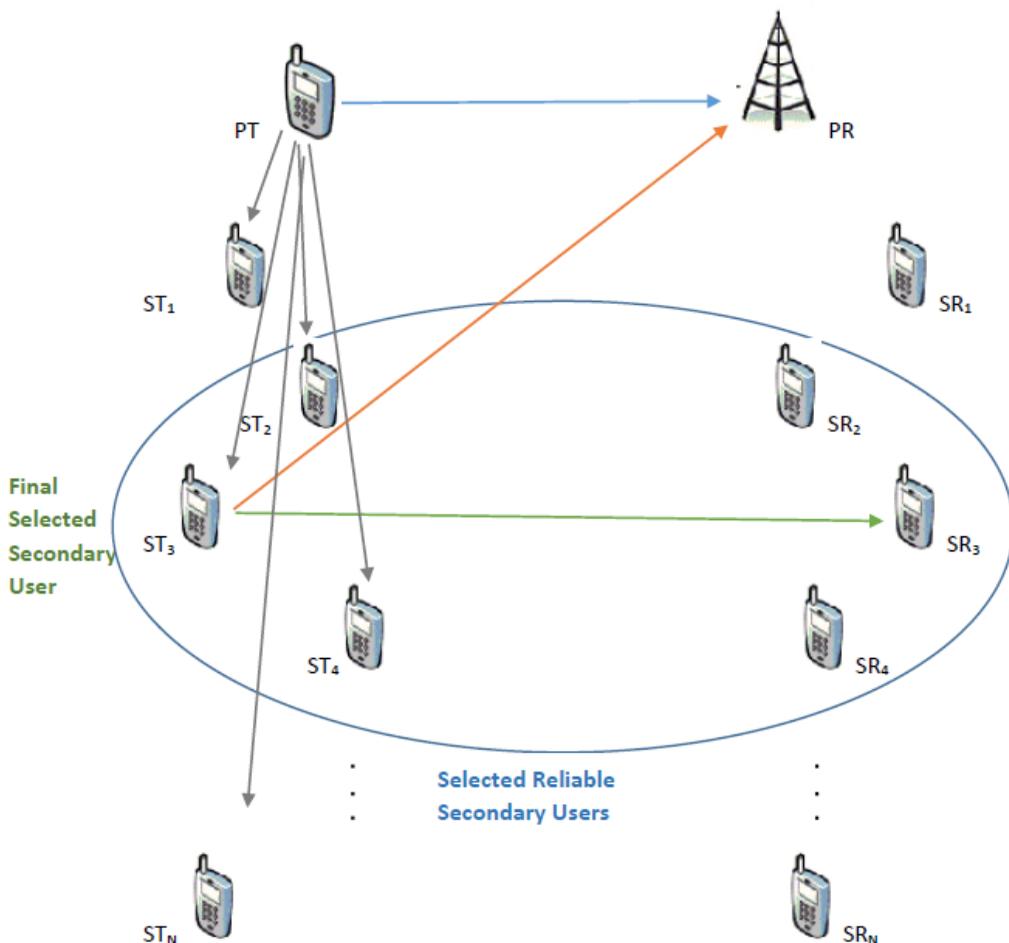
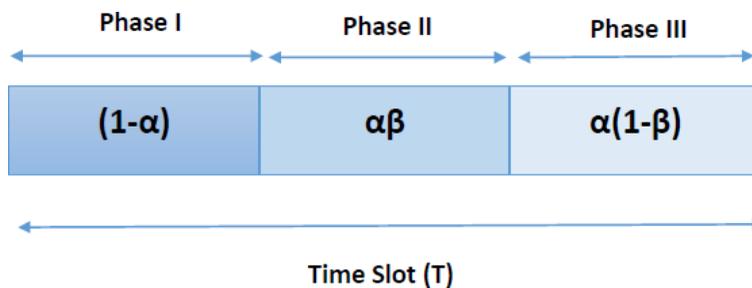


Figure 2. Time frame allocations



$h_{S_i}$  : Channel coefficient between  $ST_i$  and  $SR_i$

The expected value of channel coefficients are denoted by  $g$  with the same subscripts. For instance, the average channel gain of the primary link is  $E\left(|h_p|^2\right) = g_p$ . As mentioned earlier, the selected SU is denoted by  $S_k$  and its corresponding channel gains are obtained by substituting subscript  $i$  by  $k$ .

The perfect channel state information about all channel gains is assumed known by the primary transmitter ((Simeone, O., Stanojev, I., Savazzi, S., Bar-Ness, Y., Spagnolini, U., & Pickholtz, R., 2008), (Etkin, R., Parekh, A., & Tse, D., 2005)). The single-sided spectral density of independent Additive White Gaussian Noise (AWGN) at the primary and SUs' receivers is shown by  $N_0$ . The Decode-and-Forward (DF) relaying method is employed at the selected SU in phase II, meaning that the secondary forwards the fully decoded messages received from the PU.

The primary node transmits with a constant power  $P_p$ . The total available energy of the selected SU,  $S_k$  is  $E_{k_{max}}$ .

$$E_k + E_{k_c} \leq E_{k_{max}} \quad (1)$$

where  $E_k$ ,  $E_{k_c}$  denote the energy of individual and cooperative transmission for the selected SU,  $S_k$ , respectively. If  $P_{k_c}$  denotes the power of  $S_k$  for forwarding the received packet from the primary to its corresponding destination  $PR$ , and  $P_k$  shows the power of its individual transmission, Equation (1) can be re-written as follows by considering the time portions allocated to the cooperative packet forwarding and individual transmission (Figure 2).

$$\alpha(1 - \beta)P_k + \alpha\beta P_{k_c} \leq \alpha P_{k_{max}} \quad (2)$$

### 3. OVERVIEW ON STACKELBERG GAMES

One class of the game models is simultaneous versus sequential games ((Osborne, M., & Rubenstein, A., 1994), (Fudenberg, D., & Tirole, J., 1994), (Amir, R., & Grilo, I., 1999)). In simultaneous games, the players make their decisions independently while they can not observe other players' actions. It is worth noting that simultaneity does not necessarily mean that the players choose their strategies at the same time, but it means that each player makes a decision when he still is not aware of the other players' actions. In contrast, sequential game refers to a class of games, where players make decisions following a predefined order, and at least some players can observe the actions of the precedent players.

*Stackelberg game* is a variant of non-cooperative sequential games, in which one of the players, the so called **leader** has the highest priority. The lower priority users are called *followers*. In a Stackelberg game, the leader declares a strategy first, then the followers rationally react to the leader's action, hence the leader has the ability to impose his strategy on the followers (Osborne, M., & Rubenstein, A., 1994), (Han, Z., Niyato, D., Saad, W., Baar, T., & Hjrungnes, A., 2011), (Amir, R., & Grilo, I., 1999), (Nie, P.Y., & Zhang, P., 2008).

The solution of the Stackelberg game is called *Stackelberg equilibrium solution*. This solution is determined through finding the optimal strategy of the leader, knowing that the followers are rational and will maximize their utilities given the leader's actions.

A basic Stackelberg game can be defined as a two-player extensive game, which assumes perfect information is available to both players (Osborne, M., & Rubenstein, A., 1994). The leader chooses an action from a set  $\mathcal{A}_1$ . Then, the follower chooses an action from a set  $\mathcal{A}_2$ , after being informed of the leader's choice. The Stackelberg equilibrium solution of this game is equivalent to solutions of the following optimization problem,

$$\max_{(a_1, a_2) \in (\mathcal{A}_1, \mathcal{A}_2)} U_1(a_1, a_2)$$

$$s.t. : a_2 \in \operatorname{argmax}_{a_2 \in \mathcal{A}_2} U_1(a_1, a_2) \quad (3)$$

where  $U_1$  and  $U_2$  denote the utility functions of leader and follower, respectively.

It is worth noting that in Stackelberg game, the advantage of being the first-mover for the leader always yields better pay offs for leader compared to the game with simultaneous moves, called *Cournot games*. The intuitive reason is that the leader knows that the follower is playing the best response in order to get at least the simultaneous move payoff by choosing the Cournot game strategy (Osborne, M., & Rubenstein, A., 1994).

#### 4. PROPOSED GAME MODEL FOR SPECTRUM LEASING TO RELIABLE SECONDARY USERS

In this section, the proposed Stackelberg game model for spectrum leasing to reliable SUs is described. In this system, the PU assigns a portion of the time slot (Phase III) to the SU for the sake of the cooperative packet forwarding performed by the secondary in Phase II. The interaction between the primary and the SUs is modeled by Stackelberg game noting the hierarchical nature of the network. The PU as the owner of spectrum is considered the game leader. In this model, the time allocation is fully authorized by the primary and it has the right to determine how to divide each time slot among the afore-mentioned three activity phases of primary and secondary users in order to maximize its own transmission rate. This is performed through setting the value of parameters  $\alpha$  and  $\beta$ .

The SUs are the followers in the game, where they observe the action of the primary and become aware of the portion of time is determined for cooperation as well as the

secondary transmission. Majority of related works in the literature did not consider the possibility of existence of malicious secondary nodes in the system, which deviate from the game rules and forward the PU's packet with a lower power rather than the power set by the game rule (Simeone, O., Stanojev, I., Savazzi, S., Bar-Ness, Y., Spagnolini, U., & Pickholtz, R., 2008) (Wang, X., Ma, K., Han, Q., Liu, Z., & Guan, X., 2012), (Stanojev, I., Simeone, O., Bar-Ness, Y., & Yu, T., 2008), (Hao, X., Cheung, M.H., Wong, V., Leung, V.C.M., 2011). A commonly used presumption is that all SUs use the same power for cooperation and their own transmissions. However, there is no guarantee to assure this integrity, since the selfish users tend to preserve their limited resources by not assigning enough power to the cooperative service. The distinction of our proposed model is designing a game model which enforces a reliable cooperative manner to the potentially selfish SUs. In this model, we consider the general and realistic assumption that the SUs can decide to set the cooperative and individual power differently.

The contribution of our proposed model is the definition of cooperative credit for each SUs, which keeps the record of its power assignment. The cooperative credit enables the PU to observe the SUs performance over the time to identify the selfish SUs. In this model, at each time slot, the primary selects the most reliable SUs with largest cooperative credits to incorporate in spectrum sharing. Hence, in the proposed reputation-based scheme, the SUs try to maintain a good reputation to have a chance of being selected by the PU in the following interactions.

The designed reputation-based spectrum sharing model performs in a distributed manner, where no central controller is required to supervise the algorithm. However, most alternative incentive-based approaches, such as pricing-based schemes, need a central controller to direct the users' interactions in trading the virtual currency (Zhong, S., Chen, J., & Yang, Y., 2003), (Ileri, O., Mau, S.C. & Mandayam, N., 2005), (Afghah, F., Razi, A., & Abedi, A., 2011).

At the first stage of the game, the *PU* sets its strategies to maximize its transmission rate. The strategy of the primary includes the time allocation parameters:  $\alpha$  and  $\beta$ , and also contains selecting one SU for cooperation. The primary first chooses the best  $K$  secondary users among the  $N$  active SUs based on the users aggregated cooperative credits until this time slot. We call this search space  $\mathcal{S}(n)$  defined as

$\mathcal{S}(n) = \operatorname{argmax}_{s \in \mathcal{S}_K} \sum_{i \in s} C_i^n$ , where  $\mathcal{S}_K$  is the collection of subsets of users with cardinality  $K$ , and  $C_i^n$  is the credit of secondary user  $i$  at time  $n$ , as defined in equation (8). Then from this candidates' pool, it selects one SU (denoted by  $S_k$ ) based on the channels condition. Hence, the strategy space of the primary is defined by  $(\alpha, \beta, \kappa)$ .

In this framework, the goal of the primary is to maximize its benefit from cooperative relaying, therefore the utility of the PU is determined as the achievable transmission rate through cooperation. At each time slot, first the primary selects its best strategies to maximize its utility as specified in (4)

$$\begin{aligned} \max_{\alpha, \beta, k} U_P(\alpha, \beta, k) &= \max_{\alpha, \beta, k} R_{cop} \\ \text{s.t. : } 0 \leq \alpha, \beta \leq 1, k &\in \mathcal{S}(n) \end{aligned} \quad (4)$$

The cooperation rate of the PU,  $R_{cop}$  in case of deploying DF relaying at the selected secondary  $S_k$  is calculated as (Host-Madsen, 2002), (Laneman, J., Tse, D., & Wornell, G., 2004):

$$R_{cop} = \min\left[(1 - \alpha)R_{PS_k}, \alpha\beta R_{S_kP}\right] \quad (5)$$

where  $(1 - \alpha)R_{PS_k}$  and  $\alpha\beta R_{S_kP}$  are the achievable rates from the PU's transmitter to the selected secondary during Phase I and from the selected secondary to the PU's receiver during

Phase II, respectively. These rates are calculated as follows:

$$R_{PS_k} = \log_2 \left( 1 + \frac{|h_{PS_k}|^2 P_P}{N_0} \right) \quad (6)$$

$$R_{S_kP} = \log_2 \left( 1 + \frac{|h_{S_kP}|^2 P_{k_c}}{N_0} \right) \quad (7)$$

At the second stage, the selected SU,  $S_k$  observes the PU's strategies and reacts to that by setting the power for cooperative relaying and its own transmission. The SU aims at maximizing its transmission rate  $R_{S_k}$ , while maintaining a good cooperative reputation.

The cooperative credit of the SUs reflects the accumulated information about their cooperative strategies during the previous time slots. The credit of the secondary  $i$  at time slot  $n$  is denoted with  $C_i^n$  and is defined on a symmetric interval  $[-C, C]$ , with negative values representing lack of cooperation, and positive values representing reliable performance in packet forwarding. The cooperative credit is updated based on the recursion rule during each time slot:

$$C_i^n = C_i^{n-1} + \Delta C_i^n, \quad n \geq 0, \quad (8)$$

assuming the initial credit of  $\Delta C_i^0$  for user  $i$ .

The change in cooperative credit at time slot  $n$  is based on the difference between the power assigned by the  $k^{th}$  secondary to cooperation at time  $n$ ,  $P_{k_c}^n$  and the power assigned for its own transmission at time  $n$ ,  $P_k^n$ . For the sake of simplicity in notations, we drop the superscript  $n$  and use the notation  $P_{k_c}$  and  $P_k$ , when the time index  $n$  is clear from the context. The change in credit during the round  $n$  of the game (time slot  $n$ ) is:

$$\Delta C_i^n = C_s (P_{k_c} - P_k - P_t); \quad n \geq 0, \quad (9)$$

where  $C_s$  ( $C_s > 0$ ) is the quantization constant step and  $P_t$  is threshold power defined as a tuning parameter. When the SU assigns a big enough power for cooperation, its cooperative credit will be increased, while it will be reduced upon selfish behavior of not allocating enough power to packet forwarding.

In one hand, the secondary user  $i$  tends to increase its own transmission power,  $P_i$  to obtain a higher transmission rate and on the other hand, it needs to devote more power to cooperation,  $P_{i_c}$  to sustain a good reputation and be selected for next rounds of the game. Therefore, considering the fixed available energy to the secondary  $i$ ,  $E_{i_c} + E_i \leq E_{i_{max}}$ , it exhausts the total available energy, meaning that

$$E_{i_c} + E_i = E_{i_{max}}, \quad (10)$$

or equivalently

$$\alpha(1-\beta)P_i + \alpha\beta P_{i_c} = \alpha P_{i_{max}}. \quad (11)$$

Hence, the power allocation of secondary user  $i$  can be fully determined by either the individual transmission power  $P_i$  or the cooperation power  $P_{i_c}$ .

The utility of the secondary user  $i$  is presented in Equation (12). This utility is designed in such a way to encounter the SUs' desire to maximize their transmission rate while also accounting for energy efficiency.

$$U_{S_i}(P_{i_c}, P_i) = \alpha(1-\beta) \log_2 \left( 1 + \frac{|h_{S_i}|^2 P_i}{N_0} \right) - \eta_1 \alpha(1-\beta) P_i - \eta_2 \alpha \beta P_{i_c} \quad (12)$$

where  $\eta_1$  and  $\eta_2$  are predefined normalizing coefficients for energy to make it comparable with transmission rate.

The summary of the proposed game model is provided in Algorithm 1.

Although a one-shot Stackelberg game is used, the definition of the accumulative cooperative credits for the SUs provides the primary with the possibility of observing the past strategies of the SUs and keeping that into account in selecting the reliable SUs. In repeated games, a stage game is played repeatedly, where the players' strategies are contingent on the previous actions. The repeated game can encourage cooperation and prevent the players misbehavior, which required to monitor the game complete strategy profile over the course of time (Mailath, G. J. & Samuelson, L., 2006), (Pedru, D.B., & Frchette, G.R., 2011), (Abreu, D., Pearce, D., & Stacchetti, E., 1990). Repeated games are utilized to model the spectrum access in cognitive radio networks (Etkin, R., Parekh, A., & Tse, D., 2005), (Niyato, D., & Hossain, E., 2008). In our proposed one-shot game, we are still able to encourage the SUs to cooperate while we only need to keep their cooperative credits rather than the entire action history.

## 5. EQUILBIUM ANALYSIS

A reputation-based Sackelberg game model was proposed in section IV for cooperative spectrum sharing in cognitive radio networks. The proposed game is a one-shot Stackelberg game, which is played over each time slot, such that the time slot  $n$  is equivalent to the round  $n$  of the game. In each round, the players take their actions sequentially. At the first stage, the

Algorithm 1. Proposed Stackelberg game procedure for spectrum sharing with cooperative reliable SUs1)

- 1)  $n = 0$ , set initial cooperative credits  $C_0^i$  for SUs,  $\forall i \in N$
- 2)  $n = 1$
- 3) Do (for time slot  $n$ )
- 4) PU sets its strategies to optimize its transmission rate by
  - a) selecting the  $K$  reliable SUs with highest cooperative credits
  - b) selecting the secondary user  $S_k$  among the  $K$  reliable ones based on the channel condition
  - c) setting the values for  $\alpha$  and  $\beta$  to maximize its utility  $U_P(\alpha, \beta, k)$  (Equation (4))
- 5) SU sets  $P_k$  and  $P_{k_c}$  to optimize (12)
- 6) update the cooperative credit  $\Delta C_k^n = C_s(P_{k_c} - P_k - P_t)$
- 7) PU updates the cooperative credit  $C_k^n = C_k^{n-1} + \Delta C_k^n$
- 8)  $n = n + 1$
- 9) Goto step 2

PU (leader) optimizes its utility noting the cooperative credit of the SUs. The primary’s strategies includes setting the parameters  $\alpha$  and  $\beta$  and also selecting the best secondary relay.

The decision of the game leader is under the assumption that the selected secondary is rational and it selects its best strategy in response to the primary’s strategy. At the second stage, the followers become aware of which SU is chosen by the primary and the values of time allocation parameters. Consequently, the optimization problem at the secondary user  $k$  is given by:

$$\begin{aligned} & \max_{P_k, P_{k_c}} U_{S_k}(P_{k_c}, P_k) \\ & s. t. : 0 \leq P_{k_c}, P_k \leq P_{k_{max}}, \\ & \alpha(1 - \beta)P_k + \alpha\beta P_{k_c} = \alpha P_{k_{max}}, \end{aligned} \tag{13}$$

At each round of the game, the Stackelberg equilibrium solution can be found by a backward-induction process. Hence, to find the Stackelberg solution we first need to calculate the best response of the selected secondary  $k$  by solving the optimization problem(13). Since

the values of parameters  $\alpha$  and  $\beta$  are set by the PU and known to the SU, the strategy of the SU can be equivalently determined by the energy of individual or cooperative transmissions.

**Theorem 1:** For any given strategies of the PU, the optimum energy of the selected SU denoted by  $E_k^*$  is unique and calculated in (14) if  $\alpha \neq 0$  and  $\beta \neq 1$ .

$$E_k^* = \begin{cases} 0, & \text{if } \frac{|h_{S_k}|^2}{N_0 \ln 2} - \eta_1 + \eta_2 > 0, \\ E_{k_{max}}, & \text{if } \frac{\frac{|h_{S_k}|^2}{N_0}}{\ln 2 \left( 1 + \frac{|h_{S_k}|^2}{N_0} \frac{E_{k_{max}}}{\eta(1-\eta)} \right)} - \eta_1 + \eta_2 < 0, \\ E_k^*, & \text{otherwise} \end{cases} \tag{14}$$

where

$$E_k^* = \frac{\alpha(1-\beta)}{\frac{|h_{S_k}|^2}{N_0}} \left[ \frac{\frac{|h_{S_k}|^2}{N_0}}{\ln 2(\eta_1 - \eta_2)} - 1 \right]. \quad (15)$$

**Proof:** The optimum strategy of the selected SU is obtained by backward-induction as a function of the optimum selected strategies of the primary. The optimum energy is calculated using KKT conditions to solve optimization problem (14).

The optimization problem in (14) represents a strictly concave function if  $\alpha \neq 0$  and  $\beta \neq 1$ , since the second derivative is always negative

$$\frac{\partial^2 U_{S_k}}{\partial P_k^2} = \frac{\alpha(1-\beta)}{\ln 2} \frac{-\frac{|h_{S_k}|^2}{N_0}}{\left(1 + \frac{|h_{S_k}|^2}{N_0} P_k\right)^2} < 0. \quad (16)$$

The special condition of  $\alpha = 0$  refers to the case when the whole time slot is used for primary transmission and  $\beta = 1$  shows the extreme scenario that the whole time slot is occupied for the secondary individual transmission.

**Theorem 2:** The optimum strategies of the PU  $\alpha^*$  and  $\beta^*$  are

$$\beta^* = \max_{\beta} \beta R_{S_k P}(\beta), \quad 0 \leq \beta \leq 1 \quad (17)$$

$$\alpha^* = \frac{R_{PS_k}}{R_{PS_k} + \beta^* R_{S_k P}(\beta^*)} \quad (18)$$

**Proof:** The optimum strategies of the primary after predicting the rational response of the secondary is obtained from

$$\begin{aligned} \max_{\alpha, \beta} U_p(\alpha, \beta) = \\ \max_{\alpha, \beta} \left\{ \min \left( (1-\alpha) R_{PS_k}, \alpha \beta R_{S_k P}(\alpha) \right) \right\} \\ s.t. \quad 0 \leq \alpha, \beta \leq 1, \quad (19) \end{aligned}$$

The strategy  $\beta$  only appears on the second term of the minimum function in (19), since it can be found separately by solving optimization problem (17).

The first term of minimum function in (19),  $(1-\alpha) R_{PS_k}$  is an increasing function of  $\alpha$ , while the second term  $\alpha \beta R_{S_k P}(\beta)$  is a decreasing function of  $\alpha$ . Hence, the maximum of PU utility is obtained when the first and second term in the minimum function are equal that

$$\text{results in } \alpha^* = \frac{R_{PS_k}}{R_{PS_k} + \beta^* R_{S_k P}(\beta^*)}.$$

Both optimization problems (17) and (18) have a unique solution, therefore the solution of the PU is unique.

## 6. NUMERICAL RESULTS

In this section, the performance of the proposed reputation-based game model in comparison with the classical Stackelberg game is presented. The following parameters are assumed in the simulation:

$$N = 20, K = 4, \eta_1 = 0.5, \eta_2 = 0.2.$$

Penalizing coefficient for energy consumption by the secondary user for its individual transmission is higher than that of the cooperation phase,  $\eta_2 < \eta_1$  to encourage the secondary users to cooperate more frequently. All the channels are Block Rayleigh Fading channels with  $g_{ij} = 1, i, j \in \{P, S_i\}$ , where the channel gain is constant during one time slot, while independent over consecutive time slots. The channel SNRs for all links are arbitrarily set to 0dB.

In the simulations, we consider a case where part of the secondary users are unreliable such that they do not follow the game rule in

performing a fair energy allocation when granted channel access. Therefore, we divide the secondary users into two groups: reliable  $\Psi = \{1, 2, \dots, \lfloor \rho N \rfloor\}$  and malicious  $\Psi^c = \{\lfloor \rho N \rfloor + 1, \dots, N\}$ , where  $\rho$  is the ratio of reliable users and  $\lfloor x \rfloor$  is the largest integer not less than  $x$ . The reliable users  $(S_k, k \in \Psi)$  follow the game rule in allocating energy between cooperation and individual transmission, while the unreliable users  $(S_k, k \in \Psi^c)$  present a malicious behavior with a predefined probability and assign maximum power to their own transmissions. The probability of violation is set to  $p_v = \%60$ , meaning that the unreliable users misbehave  $\%60$  of time if chosen by the primary user.

In order to emphasize on the crucial impact of the proposed credit based solution, we compare the proposed solution with the standard Stackelberg solution, without considering credit-history of users. As detailed in section IV, in the proposed method, the search subspace  $\mathcal{S}(n)$  in time slot  $n$  is defined based on the

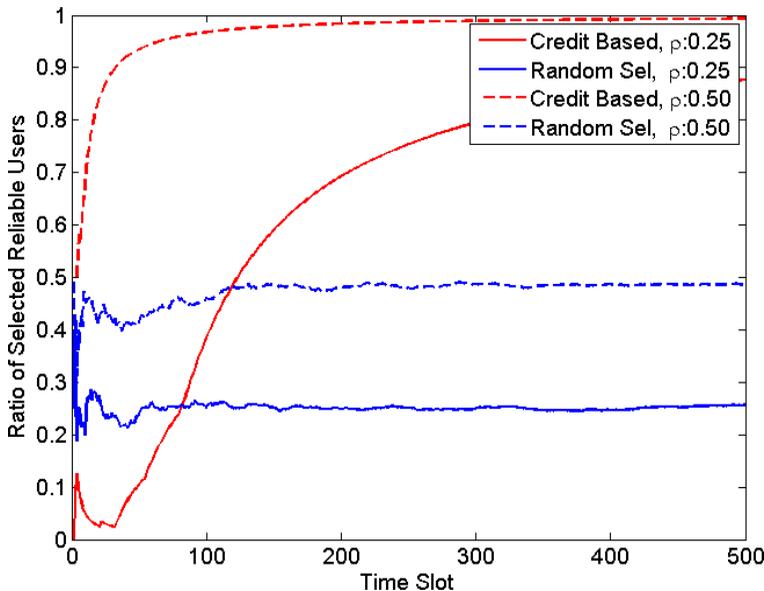
accumulated credit of users to include the most  $K$  credible users in the optimization; while in the standard Stackelberg game they are chosen randomly.

Figure 3 demonstrates the ratio of the reliable users obtained from two methods, the proposed reputation-based model (red curves) and the classical model without credit (blue curves). In this Figure, the average ratio of trusted users at time slot  $n$  means the ratio of number of trusted users remain in search space until the current time slot (i.e.

$$\frac{\sum_{i=1}^n \sum_{k \in \Psi} 1(k \in \mathcal{S}(i))}{nK}$$

It is noticeable that over first few time slots this ratio fluctuates between 0 and 1. After a few time slots, this ratio approaches to the ratio of trusted users (i.e.  $\rho$ ) for classical Stackelberg game as expected, since the random selection of  $K$  users yield the same statistical behavior as the all  $N$  user pool. However, the proposed credit based game, identifies the untrusted users and hence gradually filters them out from the search space. This is more interesting, when the untrusted users do not behave deterministi-

Figure 3. Time evolution of average ratio of trusted user selection



cally and violate the game rule with some probability. Therefore, the ratio of trusted users in the search space approach one as time evolves.

The advantage of the proposed solution from the primary perspective is demonstrated in Figure 4. This Figure compares the utility of primary user for the proposed credit based solution with the classical Stackelberg game. We see that the average utility of the primary user for the proposed game is considerably higher than the classical solution. As reasoned above, this is due to excluding the unreliable users from the system. Since these users violate the game rule and avoid cooperative relaying that results in reducing the average primary user's utility. Our proposed game model encourages the secondary users to follow the game rule, since otherwise they will not be selected as the reliable nodes in the subsequent time slots to obtain bandwidth access.

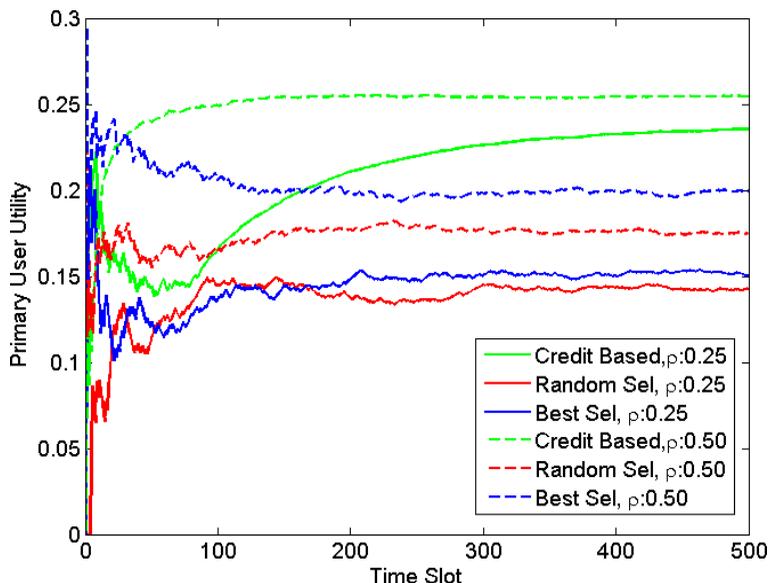
## 7. CONCLUSION

In this paper, a reputation-based Stackelberg game model for spectrum leasing in a CRN is

presented. In this model, a single primary and multiple potentially dishonest SUs coexist in the network. At each time slot, the primary assigns a portion of time to only one SU considering the cooperative credit and the channel quality. A cooperative credit is defined for each secondary based on the amount of power it assigns to relay the PU's packet. This mechanism encourages the SUs to maintain a good reputation to obtain the chance of spectrum access.

The proposed framework enables the primary to recognize malicious SUs and only interact with the reliable nodes among all active SUs. As shown in numerical results, the proposed model successfully recognize the unreliable users and will not consider them for cooperative spectrum sharing in future time slots. This solution has the advantage of i) lower signaling overhead due to reduced search space, ii) higher primary utility by avoiding dishonest user selection for cooperation and iii) higher chances for reliable secondary users to get channel access by filtering out the dishonest users.

Figure 4. Comparison of primary user utility for Stackelberg game with and without considering credit history



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