Inference in first-order logic

Chapter 9

Outline

- ✦ Reducing first-order inference to propositional inference
- ✦ Unification
- ✦ Generalized Modus Ponens
- ✦ Forward and backward chaining
- ✦ Logic programming
- ✦ Resolution

Reasoning in FOL

- We know how to efficiently represent complex worlds in FOL
  - Quantification + variables → FOL much more flexible, compact
- We know how to do inference in propositional logics
  - Model-checking OR inference rules OR resolution
  - Can be arduous, but it’s fairly mechanical. Sound and complete
  - Size of problem becomes impossible as complexity of world grows.
- Would like to use FOL...and do inference in FOL.
  - Quantification + variables → make things much more complex.
- Question: How can we reason in FOL?
  - We can convert FOL to propositional → use propositional inferencing
  - We can develop new ideas to deal with instantiation/variables
    - Reasoning directly in FOL!

Universal Instantiation (UI)

- Goal: Get rid of universal quantifiers
- Plan: Logically equivalent replacement
  - Consider all facts that a universally quantified sentence could imply.
  - Replace the quantified sentence by all implied facts.

- Example. Consider:
  - $\forall x \; King(x) \land Greedy(x) \Rightarrow Evil(x)$

- What are all the *literal* facts that this could imply?
  - $King(John) \land Greedy(John) \Rightarrow Evil(John)$
  - $King(Richard) \land Greedy(Richard) \Rightarrow Evil(Richard)$
  - $King(Father(John)) \land Greedy(Father(John)) \Rightarrow Evil(Father(John))$
  - Etc. etc. etc. → "$\forall x$" means "true for every possible literal substitution

- Every possible instantiation of a universally quantified sentence is entailed by it: $\dfrac{\forall v \; \alpha}{Subst(\{v/g\}, \alpha)}$ for any variable v and ground term g
- So: for a given FOL fact base: substitute all $\forall x$ with literal facts implied.

Existential Instantiation (EI)

- Goal: Get rid of existential quantifiers
- Plan: Logically equivalent replacement
- Example. Consider:
  - $\exists x$ Crown(x) $\wedge$ OnHead(x, John)

- What exactly does this *mean*?
  - There is *some* literal in the world for which this is true.
  - But for which literal? What if we don't know it yet? Or anonymous: mom(Kate)

- Answer: create a special unique anonymous literal: Skolem Constant

$$\frac{\exists v \quad \alpha}{\mathrm{Subst}(\{v/k\}, \alpha)}$$

For any sentence α, variable v, and constant symbol k that does not appear elsewhere in the knowledge base

  - So: Crown(C1) $\wedge$ OnHead(C1, John), where C1 is a new Skolem symbol

- So, to get rid of quantification:
  - UI can be applied several times to add new sentences
    - the new KB is logically equivalent to the old
  - EI can be applied once to replace the existential sentence
    - the new KB is not strictly equivalent to the old...but is satisfiable iff the old KB was satisfiable

Example: Reduction to Propositional Inference

- Suppose the KB contains just the following:
  - $\forall x\ King(x) \land Greedy(x) \Rightarrow Evil(x)$
  - King(John)
  - Greedy(John)
  - Brother(Richard, John)

- Instantiating the universal sentence in all possible ways, we have
  - $King(John) \land Greedy(John) \Rightarrow Evil(John)$
  - $King(Richard) \land Greedy(Richard) \Rightarrow Evil(Richard)$
  - King(John)
  - Greedy(John)
  - Brother(Richard, John)

- The new KB is propositionalized:
  - proposition symbols: King(John), Greedy(John), Evil(John), King(Richard) etc.
  - Note: not predicates! Consider them monolithic symbols like $B_{1,1}$ and $W_{3,2}$

Analysis: Reducing FOL to Propositional Inference

- Claim: a sentence α is entailed by new KB iff entailed by original KB
- Claim: every FOL KB can be propositionalized so as to preserve entailment
- Idea! Do inference in FOL KB by:
  1. Propositionalize KB and query
  2. apply propositional resolution, return result
- Problem: function symbols: for any input, yield an output entity
  - Can refer to infinitely many entities, e.g., Father(Father(Father(John)))
  - Propositionalized KB is essentially infinite!

- Theorem: Herbrand (1930):
  - If a sentence α is entailed by an FOL KB, it is entailed by a finite subset of the propositional KB

- Yields new Idea: For n = 0 to ∞ do
  - create a propositional KB by instantiating with depth-n terms
  - see if α is entailed by this KB
- Basically the IDS concept applied to proving entailment

- Problem: works if α is entailed... But never stop looking if α is not entailed
  - Theorem: Turing (1936), Church (1936), entailment in FOL is semidecidable

Other practical problems with propositionalization

- Another problem: Propositionalization generates lots of irrelevant sentences.
  - E.g., from:
    - $\forall x$ King(x) $\wedge$ Greedy(x) $\Rightarrow$ Evil(x)
    - King(John), Prince(Andrew), Lady(Di), Stodgy(Mum)
    - $\forall y$ Greedy(y)
    - Brother(Richard, John)
  - it seems instantly *obvious* to human readers that Evil(John),

    ...but propositionalization produces *lots* of facts that are irrelevant:
    - Greedy(Richard), Greedy(Andrew), Greedy(Di), Greedy(Mum), etc
- With p k-ary predicates and n constants, there are $p \cdot n^k$ instantiations!
  - And with function symbols, it gets much much worse!
- We need a more straightforward way to inference directly in FOL !!!

Unification

- Observation:
  - Given previous KB, we can get the inference Evil(John) immediately... if we can find a substitution θ such that King(x) and Greedy(x) match King(John) and Greedy(y)
- θ = {x/John, y/John} works!
- Definition: Unify(α, β) = θ if subst(θ, α) = subst(θ, β)
  - Two terms α and β *unify*, if you can find a variable binding θ that, when substituted in, makes the terms identical
- Example: Unify the following sentences:

p	q	θ
$Knows(John, x)$	$Knows(John, Jane)$	
$Knows(John, x)$	$Knows(y, OJ)$	
$Knows(John, x)$	$Knows(y, Mother(y))$	
$Knows(John, x)$	$Knows(x, OJ)$	

- The last one fails...but shouldn't. What's going on?
  - The 'x' in Knows(John,x) is not necessarily same as 'x' in Knows(x,OJ)
    - Both are just variables...could be bound to anything.
  - Standardize apart: Variables in each clause made unique: Knows(John, x_{217})

Generalized Modus Ponens (GMP)

- We can leverage unification to generalize Modus Ponens Inference!
  - Reminder: Modus ponens was

$$\frac{\alpha \Rightarrow \beta,\ \alpha}{\beta}$$

Or, more generally:

$$\frac{p_1, p_2, p_3 \ldots p_n,\ (p_1 \land p_2 \land \ldots \land p_n) \Rightarrow q)}{q}$$

  - But now suppose that p_i and q are all parameterized by variables
  - If there is some set of bindings θ that makes all sentences in the premise of an implication identical to sentences already in the KB, then we can assert the conclusion.
  - Or formally: $$\frac{p'_1, p'_2, p'_3 \ldots p'_n,\ (p_1 \land p_2 \land \ldots \land p_n) \Rightarrow q)}{\mathrm{subst}(\theta, q)}$$ where $\mathrm{subst}(\theta, p_i') = \mathrm{subst}(\theta, p_i)$ for all i
  - So then: $$\frac{\mathrm{King(John)}, \mathrm{Greedy}(y),\ (\mathrm{King}(x) \land \mathrm{Greedy}(y)) \Rightarrow \mathrm{Evil}(x))}{\mathrm{Evil(John)}}$$ for θ={x/John, y/John}

- Note that GMP is used with:
  - KB of definite clauses (exactly one positive term)
  - All variables are assumed to be universally quantified

Example: Putting it together

The law says that it is a crime for an American to sell weapons to hostile nations. The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American.

Prove that Col. West is a criminal

Example: Putting it together (cont.)

. . . it is a crime for an American to sell weapons to hostile nations:

Example: Putting it together (cont.)

. . . it is a crime for an American to sell weapons to hostile nations:

$$American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x)$$

Nono . . . has some missiles

Example: Putting it together (cont.)

. . . it is a crime for an American to sell weapons to hostile nations:

$American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x)$

Nono . . . has some missiles, i.e., $\exists\, x\; Owns(Nono, x) \wedge Missile(x)$:

$Owns(Nono, M_1)$ and $Missile(M_1)$

. . . all of its missiles were sold to it by Colonel West

Example: Putting it together (cont.)

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$American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x)$

Nono . . . has some missiles,

i.e., $\exists\, x\ Owns(Nono, x) \wedge Missile(x)$...and applying EI

$Owns(Nono, M_1)$ and $Missile(M_1)$

. . . all of its missiles were sold to it by Colonel West

$\forall x\ \ Missile(x) \wedge Owns(Nono, x) \quad \Rightarrow \ Sells(West, x, Nono)$

Missiles are weapons:

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$\forall\; x\; Missile(x) \wedge Owns(Nono, x) \quad \Rightarrow \; Sells(West, x, Nono)$

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$Missile(x) \Rightarrow Weapon(x)$

An enemy of America counts as "hostile":

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Missiles are weapons:

$Missile(x) \Rightarrow Weapon(x)$

An enemy of America counts as "hostile":

$Enemy(x, America) \Rightarrow Hostile(x)$

West, who is American . . .

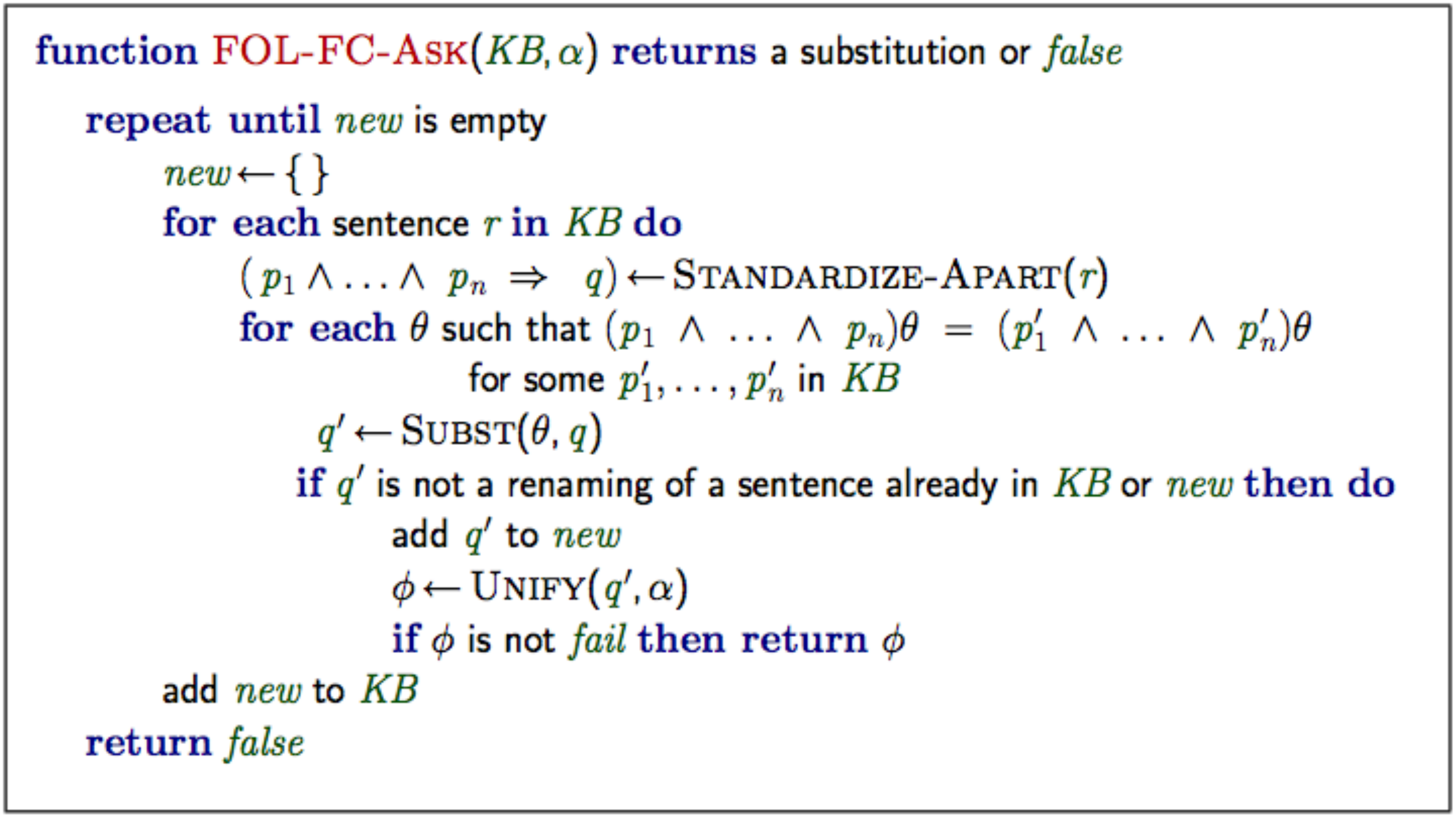
$American(West)$

The country Nono, an enemy of America . . .

$Enemy(Nono, America)$

Inference algorithms: Forward chaining

Similar to propositional logic, we can infer new facts using forward chaining



```
function FOL-FC-ASK($KB, \alpha$) returns a substitution or $false$
    repeat until $new$ is empty
        $new \leftarrow \{\}$
        for each sentence $r$ in $KB$ do
            $(p_1 \land \ldots \land p_n \Rightarrow q) \leftarrow$ STANDARDIZE-APART($r$)
            for each $\theta$ such that $(p_1 \land \ldots \land p_n)\theta = (p'_1 \land \ldots \land p'_n)\theta$
                        for some $p'_1, \ldots, p'_n$ in $KB$
                $q' \leftarrow$ SUBST($\theta, q$)
                if $q'$ is not a renaming of a sentence already in $KB$ or $new$ then do
                    add $q'$ to $new$
                    $\phi \leftarrow$ UNIFY($q', \alpha$)
                    if $\phi$ is not $fail$ then return $\phi$
        add $new$ to $KB$
    return $false$
```

Forward chaining proof

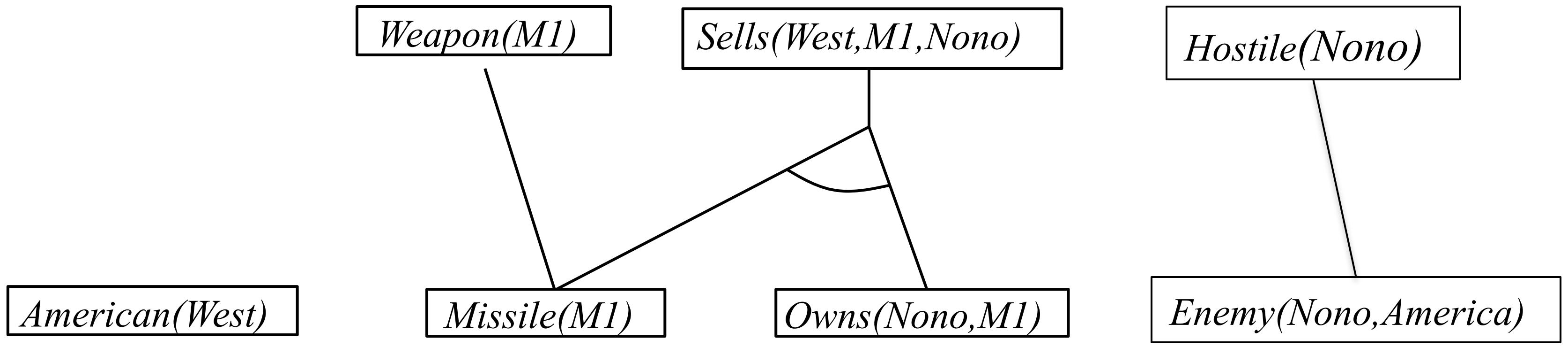
First we have the known facts...

$\boxed{\mathit{American(West)}}$ $\boxed{\mathit{Missile(M1)}}$ $\boxed{\mathit{Owns(Nono,M1)}}$ $\boxed{\mathit{Enemy(Nono,America)}}$

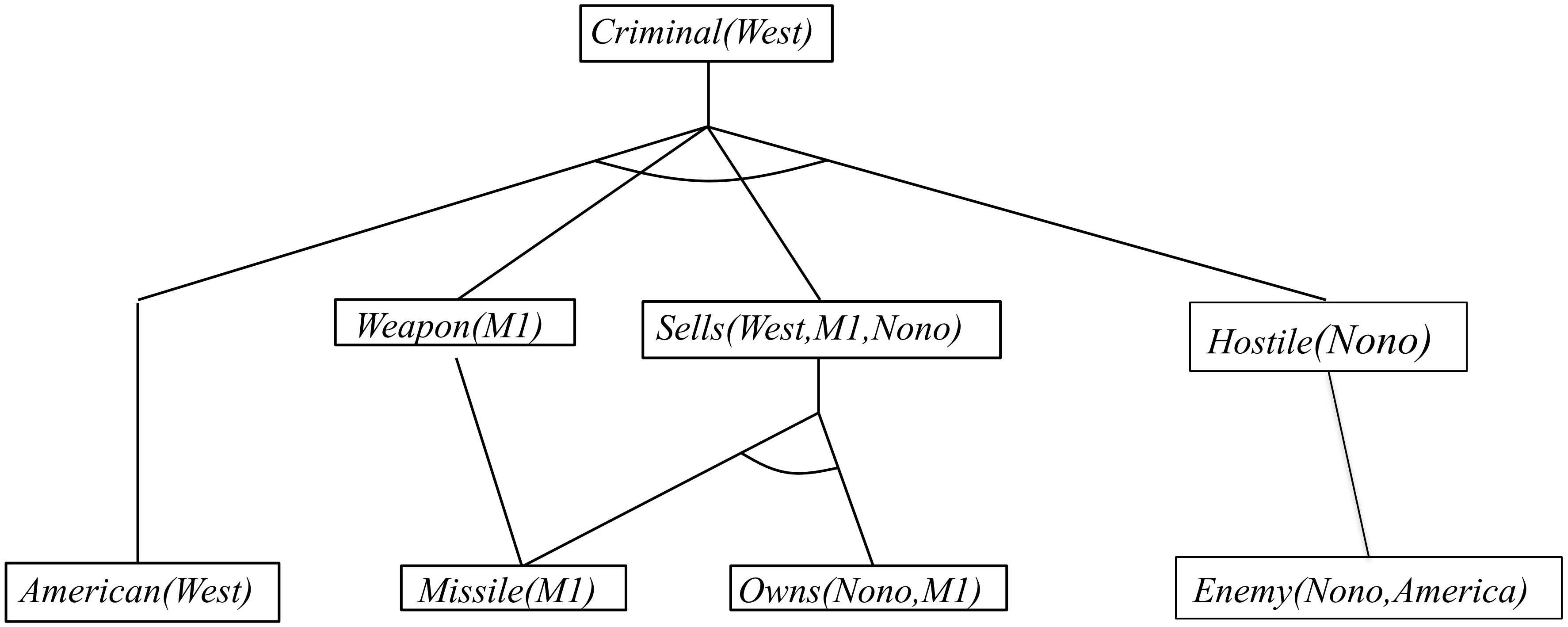
Forward chaining proof

Forward chaining with:

- $\forall x\ \mathit{Missile}(x) \land \mathit{Owns}(\mathit{Nono}, x) \Rightarrow \mathit{Sells}(\mathit{West}, x, \mathit{Nono})$
- $\mathit{Missile}(x) \Rightarrow \mathit{Weapon}(x)$
- $\mathit{Enemy}(x, \mathit{America}) \Rightarrow \mathit{Hostile}(x)$



Forward chaining proof



Continuing to infer with:

- $American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x)$

Forward Chaining Proof

- Properties of Forward-chaining proof
  - Sound and complete for first-order definite clauses
    - Proof similar to propositional proof.
  - For Datalog (first-order definite clauses + no functions), FC terminates for Datalog in poly iterations
    - There at most $p \cdot n^k$ literals = upper bound on number of iterations.
    - May not terminate in general if α is not entailed. Fn's generate infinite new facts.
    - This is unavoidable: entailment with definite clauses is semi-decidable
- Efficiency of forward chaining proof. Sources of inefficiency
  - Matching conjunctive premises to *all possible* literals
    - Can often *order* conjunctive clauses to quickly constrain literals.
  - Redundant matching of all rules on each cycle
    - Simple observation: no need to match a rule on iteration k if a premise wasn't added on iteration $k - 1$ $\Rightarrow$ match *only those rules* whose premise contains a newly added literal
  - Waste time generating facts irrelevant to goal
    - Inherent to forward chaining. Backward chaining more efficient for goal-directed.
    - Magic sets: Can dynamically rewrite rules to "specialize" them toward desired goal

Backward chaining algorithm

For query-oriented (goal-directed) proofs, backward chaining is more natural

- Note that the algorithm is depth-first, recursive.

```
function FOL-BC-Ask(KB, goals, θ) returns a set of substitutions
    inputs: KB, a knowledge base
            goals, a list of conjuncts forming a query (θ already applied)
            θ, the current substitution, initially the empty substitution { }
    local variables: answers, a set of substitutions, initially empty

    if goals is empty then return {θ}   ;; if I've proven everything, I'm done
    q' ← Subst(θ, First(goals))   ;; else grab the first goal in list and try to prove
    for each sentence r in KB
            where Standardize-Apart(r) = ( p1 ∧ . . . ∧ pn ⇒ q)
            and θ' ← Unify(q, q') succeeds   ;; unify goal with head of rule
        new goals ← [ p1, . . . , pn|Rest(goals)]   ;; add RHS of rule to goal stack
        ;; now plug latest bindings made into goal stack and call recursively...
        answers ← FOL-BC-Ask(KB, new goals, Compose(θ', θ)) ∪ answers
    return answers
```

Returns a set of substitutions

- All possible sets of bindings for which entailment holds
- Could also return an "iterator": new set of bindings each time you ask

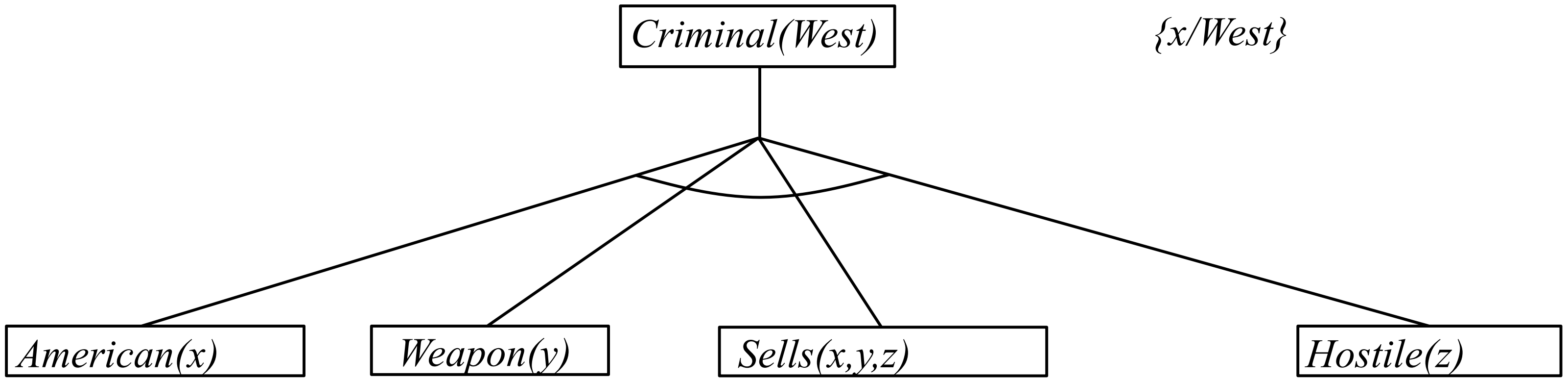
Backward chaining example

“Prove that Colonel West is a criminal”

$\boxed{Criminal(West)}$

$American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x)$
$Owns(Nono, M_1)$ and $Missile(M_1)$
$\forall x \quad Missile(x) \wedge Owns(Nono, x) \Rightarrow Sells(West, x, Nono)$
$Missile(x) \Rightarrow Weapon(x)$
$Enemy(x, America) \Rightarrow Hostile(x)$
$American(West)$
$Enemy(Nono, America)$

Backward chaining example



$American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x)$

$Owns(Nono, M_1)$ and $Missile(M_1)$

$\forall x \quad Missile(x) \wedge Owns(Nono, x) \Rightarrow Sells(West, x, Nono)$

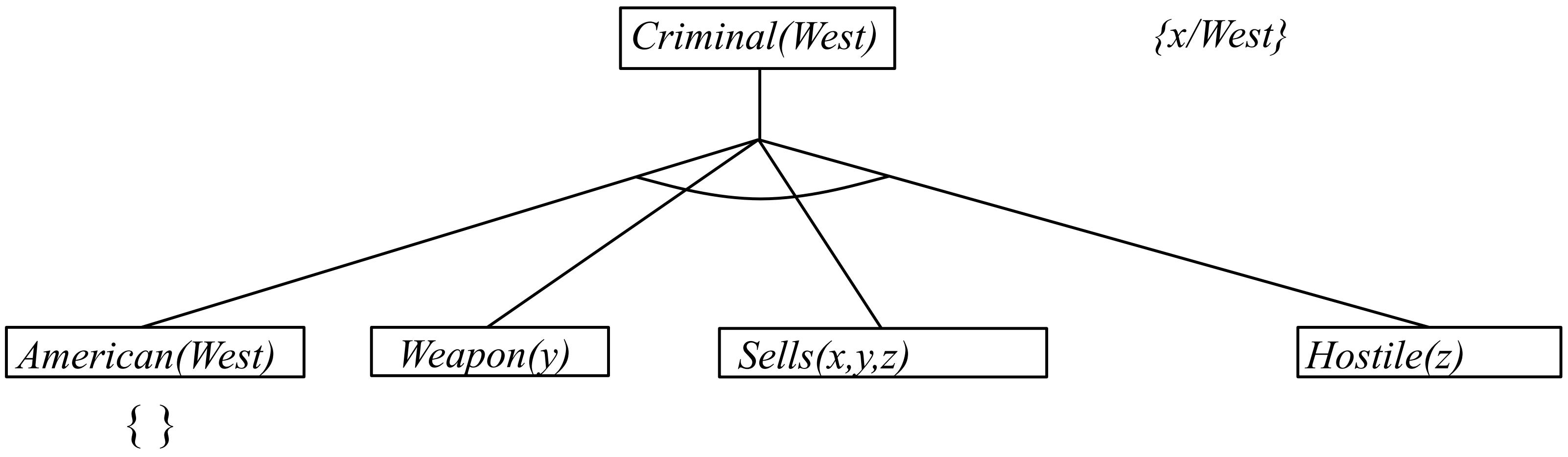
$Missile(x) \Rightarrow Weapon(x)$

$Enemy(x, America) \Rightarrow Hostile(x)$

$American(West)$

$Enemy(Nono, America)$

Backward chaining example



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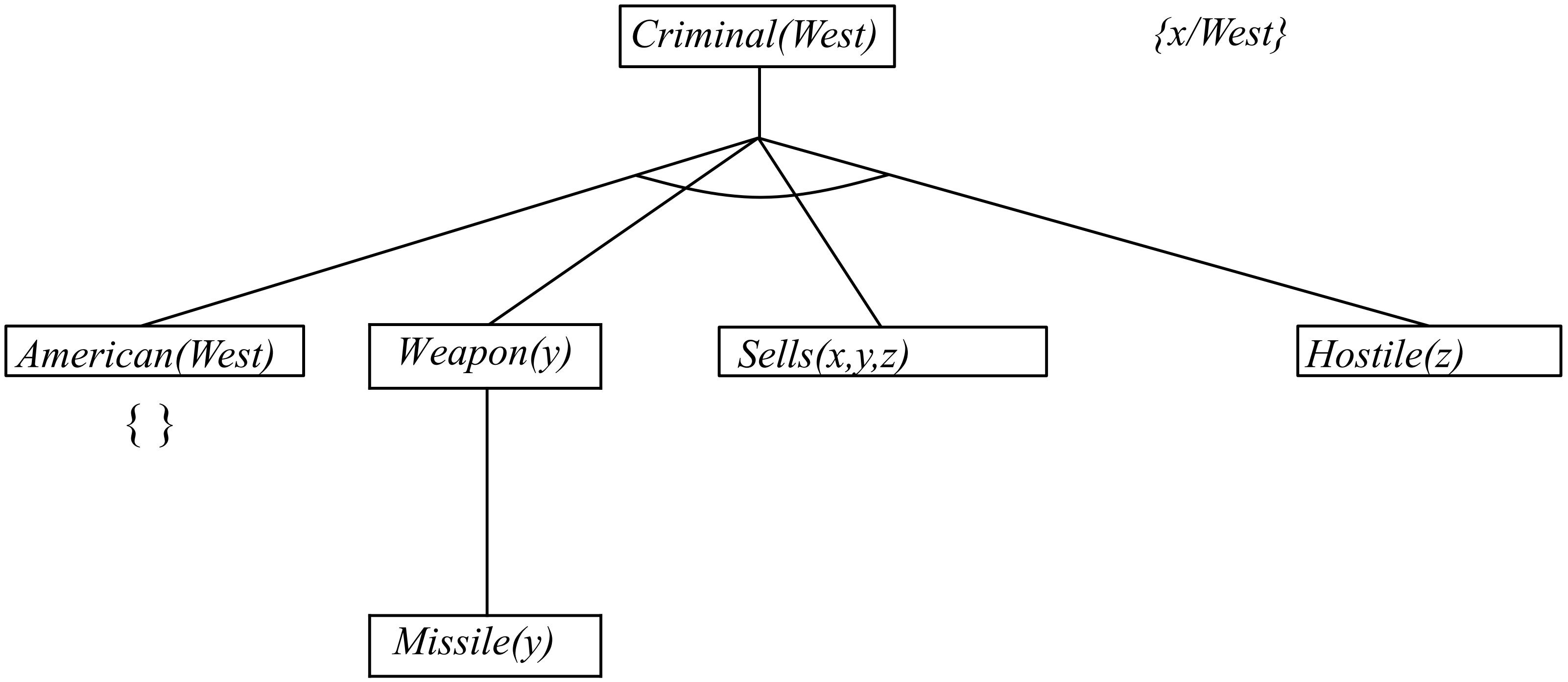

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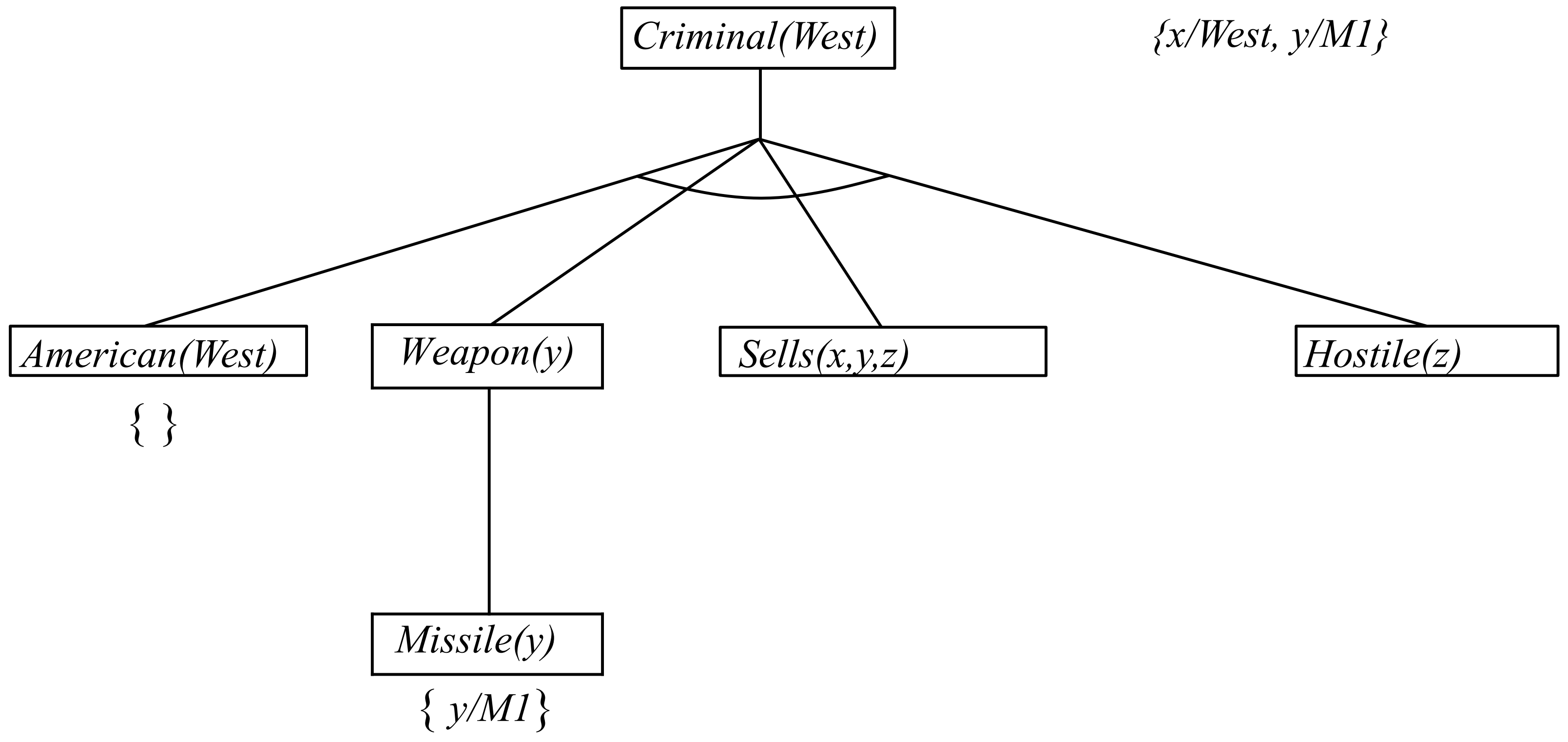

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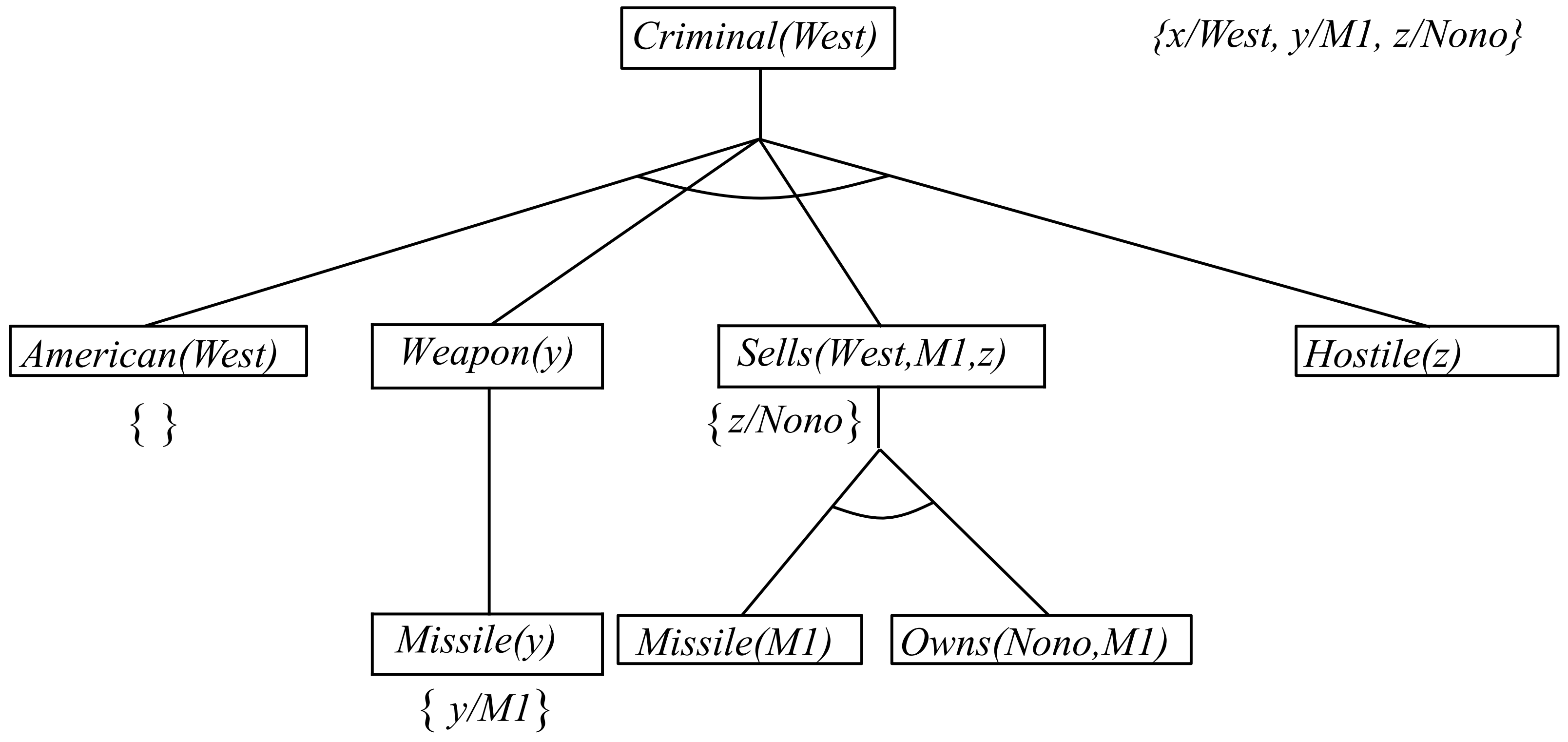

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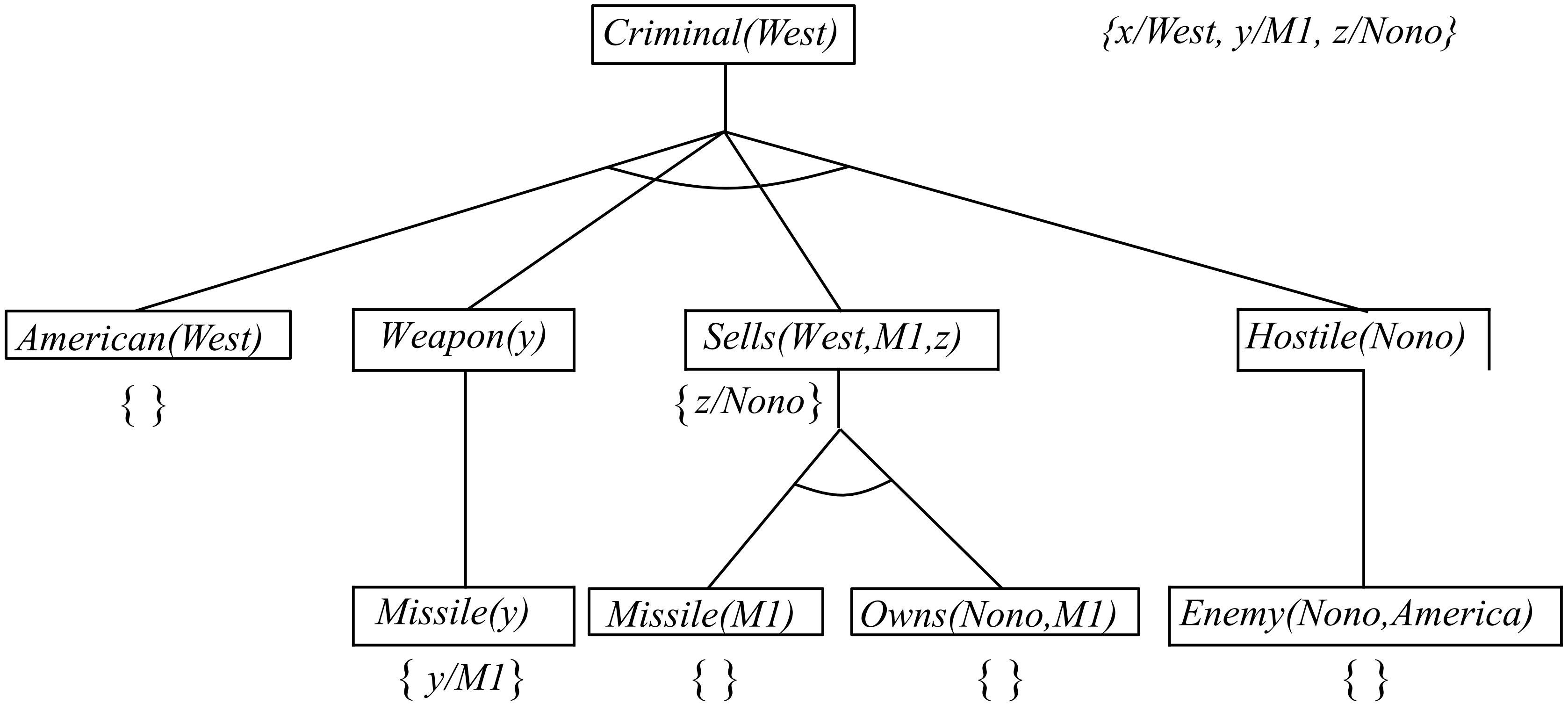
Backward chaining example



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$American(West)$
$Enemy(Nono, America)$

Backward Chaining Proof

- Properties of Backward chaining proof
  - Depth-first recursive proof search
    - space is linear in size of proof
  - Incomplete due to infinite loops
    - ⇒ fix by checking current goal against every goal on stack
    - Breaks loop...but may not solve incompleteness problem (inherent to DFS)
  - Inefficient due to repeated subgoals (both success and failure)
    - fix using caching of previous results (extra space!)
  - Widely used (with improvements!) for logic programming

Logic programming

Logic programming:

- is a way to solve problems declaratively
- Based directly on FOL... (using datalog semantics)

Conceptually: computation as inference on logical KBs

Logic programming

1. Identify problem
2. Assemble information
3. Tea break
4. Encode information in KB
5. Encode problem instance as facts
6. Ask queries
7. Find false facts

Ordinary Procedural Programming

1. Identify problem
2. Assemble information
3. Figure out solution
4. Program solution
5. Encode problem instance as data
6. Apply program to data
7. Debug procedural errors

Should be easier to debug $Capital(NewYork, US)$ than $x := x + 2$!

Prolog systems

- Basis: backward chaining with Horn clauses + bells & whistles
- Widely used in Europe, Japan (basis of 5th Generation project)
  - Compilation techniques ⇒ approaching a billion LIPS
- Program = set of clauses = head :- literal$_1$, . . . literal$_n$.
  - e.g.: criminal(X) :- american(X), weapon(Y), sells(X,Y,Z), hostile(Z).
- Depth-first, left-to-right backward chaining
- Built-in predicates for arithmetic etc.
  - e.g.: X is Y*Z+3
- Closed-world assumption (“negation as failure”)
  - e.g.: given alive(X) :- not dead(X).
  - alive(joe) succeeds if dead(joe) fails

Prolog examples

- Depth-first search from a start state X:

```
dfs(X) :- goal(X).
dfs(X) :- successor(X,S), dfs(S).
```

  - No need to loop over S: successor succeeds for each

- Appending two lists to produce a third:

```
append([],Y,Y).
append([X|L],Y,[X|Z]) :- append(L,Y,Z).
```

  - Query: append([2],[4,6,8],Z).
    - Yields (yawn): [2,4,6,8]
  - query: append(A,B,[1,2]) ?
    - WOW!! Much more powerful than procedural functions!
    - Provides all possible bindings of A, B that could (when appended) yield [1,2]
    - A=[] B=[1,2] ; A=[1] B=[2] ; A=[1,2] B=[]

FOL Resolution: Brief summary

- Principle: Identical to propositional resolution
- Full first-order version:

$$\frac{l_1 \vee \ldots \vee l_k, \quad m_1 \vee \ldots \vee m_n}{\mathrm{Subst}(\theta, (l_1 \vee \ldots \vee l_{i-1} \vee l_{i+1} \vee \ldots \vee l_k \vee m_1 \vee \ldots \vee m_{j-1} \vee m_{j+1} \vee \ldots \vee m_n)}$$

  – where $\mathrm{Unify}(l_i, \neg m_j) = \theta$.

- So same as before: match complemented predicates
  – But now: that unify with some set of bindings to be identical
- For example,

$$\frac{\neg \mathrm{Rich}(x) \vee \mathrm{Unhappy}(x) \quad \mathrm{Rich}(Ken)}{\mathrm{Unhappy}(Ken)}$$

  – with $\theta = \{x/Ken\}$

- To show entailment of α : Apply resolution steps to CNF (KB ∧ ¬α)
- Is complete for FOL

Conversion to CNF

Everyone who loves all animals is loved by someone:

$$\forall x\,[\forall y\, Animal(y) \Rightarrow Loves(x,y)] \Rightarrow [\exists y\; Loves(y,x)]$$

1. Eliminate biconditionals and implications

$$\forall x\,[\neg\forall y\; \neg Animal(y) \vee Loves(x,y)] \vee [\exists y\, Loves(y,x)]$$

2. Move $\neg$ inwards:

- Note that: $\neg\forall x, p \equiv \exists x\, \neg p$
- And that: $\neg\exists x, p \equiv \forall x\, \neg p$

So:

$$\forall x\quad [\exists y\quad \neg(\neg Animal(y) \vee Loves(x,y))] \vee [\exists y\;\; Loves(y,x)]$$
$$\forall x\quad [\exists y\quad \neg\neg Animal(y) \wedge \neg Loves(x,y)] \vee [\exists y\;\; Loves(y,x)]$$
$$\forall x\quad [\exists y\quad Animal(y) \wedge \neg Loves(x,y)] \vee [\exists y\;\; Loves(y,x)]$$

Conversion to CNF (contd.)

3. Standardize variables: each quantifier should use a different one

$$\forall x \ [\exists\, y\ Animal(y) \land \neg Loves(x, y)] \lor [\exists\, z\ Loves(z, x)]$$

4. Skolemize:
   - a more general form of existential instantiation.
   - Each existential variable is replaced by a Skolem function of the enclosing universally quantified variables:
     - Could generate any possible literal as the one that “exists”

$$\forall x \ [Animal(F(x)) \land \neg Loves(x, F(x))] \lor Loves(G(x), x)$$

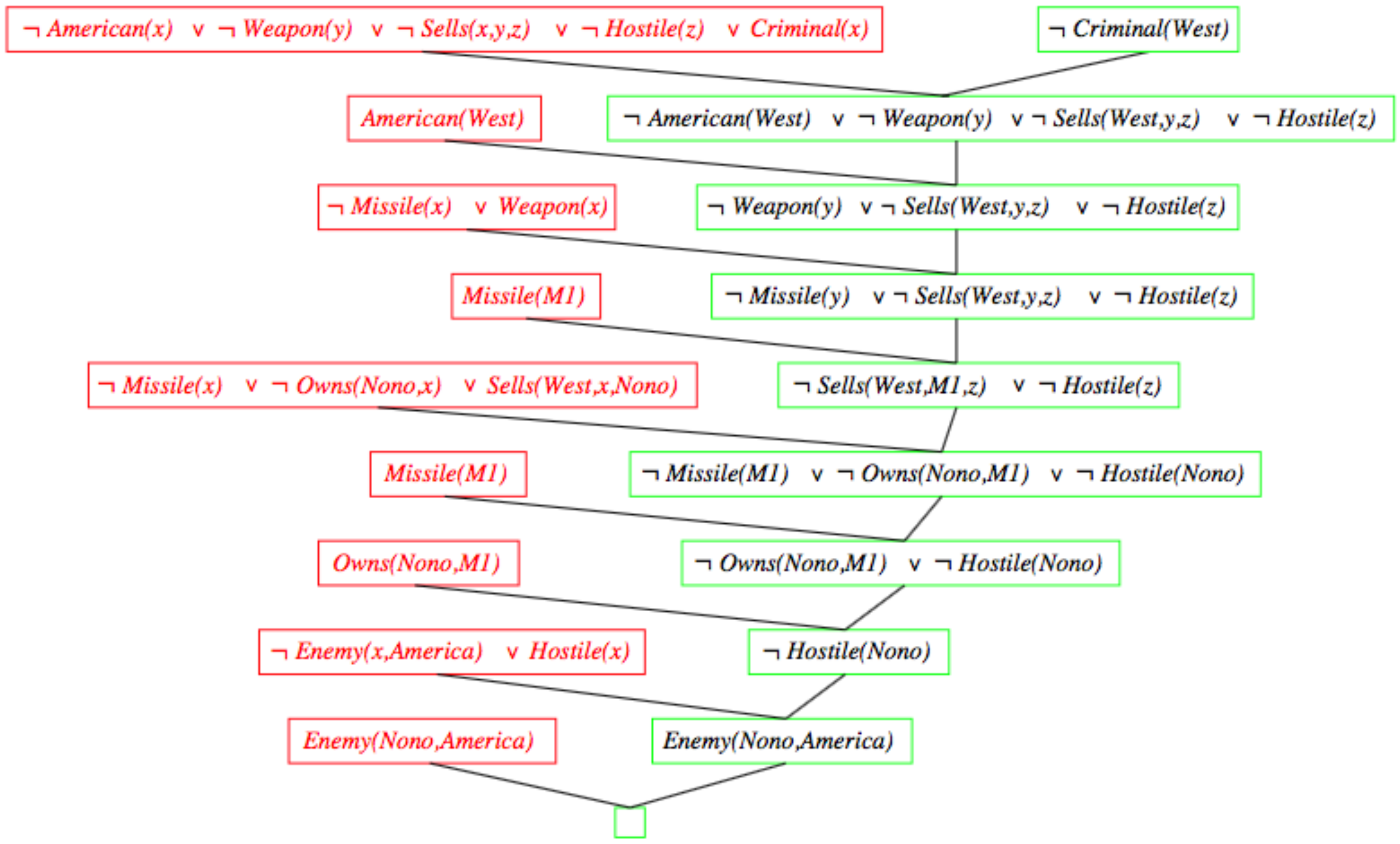
5. Drop universal quantifiers:

$$[Animal(F(x)) \land \neg Loves(x, F(x))] \lor Loves(G(x), x)$$

6. Distribute $\land$ over $\lor$:

$$[Animal(F(x)) \lor Loves(G(x), x)] \land [\neg Loves(x, F(x)) \lor Loves(G(x), x)]$$

Example: Resolution proof


¬ American(x) ∨ ¬ Weapon(y) ∨ ¬ Sells(x,y,z) ∨ ¬ Hostile(z) ∨ Criminal(x)
¬ Criminal(West)
American(West)
¬ American(West) ∨ ¬ Weapon(y) ∨ ¬ Sells(West,y,z) ∨ ¬ Hostile(z)
¬ Missile(x) ∨ Weapon(x)
¬ Weapon(y) ∨ ¬ Sells(West,y,z) ∨ ¬ Hostile(z)
Missile(M1)
¬ Missile(y) ∨ ¬ Sells(West,y,z) ∨ ¬ Hostile(z)
¬ Missile(x) ∨ ¬ Owns(Nono,x) ∨ Sells(West,x,Nono)
¬ Sells(West,M1,z) ∨ ¬ Hostile(z)
Missile(M1)
¬ Missile(M1) ∨ ¬ Owns(Nono,M1) ∨ ¬ Hostile(Nono)
Owns(Nono,M1)
¬ Owns(Nono,M1) ∨ ¬ Hostile(Nono)
¬ Enemy(x,America) ∨ Hostile(x)
¬ Hostile(Nono)
Enemy(Nono,America)
Enemy(Nono,America)

Summary: Inference in FOL

- Fred Flintstone: Turn FOL into propositional logic; find entailments
  - Use of Universal Instantiation and Existential Instantiation
  - Works...but is slow for all but the smallest domains.

- Better plan: Use unification to identify possible bindings of variables!
  - Can be seen as a “goal-focused instantiation”
  - Then can just use generalized Modus Ponens for inference
  - Forward and Backward chaining algorithms implement this approach in two different ways

- Forward chaining is used in production systems (expert systems)
  - Forward chaining is complete for datalog → runs in polynomial time

- Backward chaining is best for query-oriented proof
  - Used in logic programming systems like Prolog
  - Suffers from typical DFS issues: infinite loops and incompleteness

- Prolog uses database semantics to implement a subset of FOL

- Generalized resolution proof provides a complete proof system for FOL

α β $\subseteq$ $\neg$ $\Rightarrow$ $\models$ $\wedge$ $\vee$
$\Leftrightarrow$