Beyond Classical Search

C h a p t e r 4

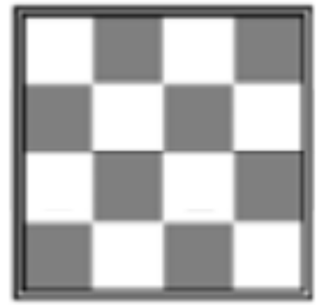
(Adapted from Stuart Russel, Dan Klein, and others. Thanks guys!)

Outline

- Hill-climbing
- Simulated annealing
- Genetic algorithms (briefly)
- Local search in continuous spaces (very briefly)
- Searching with non-deterministic actions
- Searching with partial observations
- Online search

Motivation: Types of problems

- Planning problems:
  - We want a path to a solution (examples?)
  - Usually want an optimal path
  - Incremental formulations



- Identification problems:
  - We actually just want to know what the goal is (examples?)
  - Usually want an optimal goal
  - Complete-state formulations
  - *Iterative improvement algorithms*

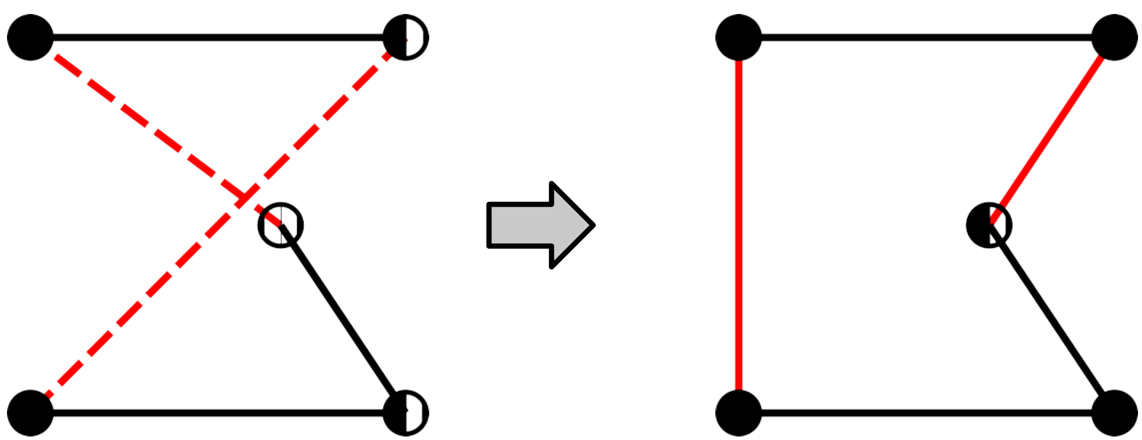
Local Search Algorithms

- So far: our algorithms explore state space methodically
  - Keep one or more paths in memory
- In many optimization problems, path is irrelevant
  - the goal state itself is the solution
  - State space is large/complex → keeping whole frontier in memory is impractical
  - Local = Zen = has no idea where it is, just immediate descendants
- State space = set of "complete" configurations
  - A graph of boards, map locations, whatever
  - Connected by actions
- Goal: find optimal configuration (e.g. Traveling Salesman)
  or, find configuration satisfying constraints, (e.g., timetable)
- In such cases, can use local search algorithms
  - keep a single "current" state, try to improve it
  - Constant space, suitable for online as well as offline search

Example: Travelling Salesperson Problem

Goal: Find shortest path that visits all graph nodes

Plan: Start with any complete tour, perform pairwise exchanges



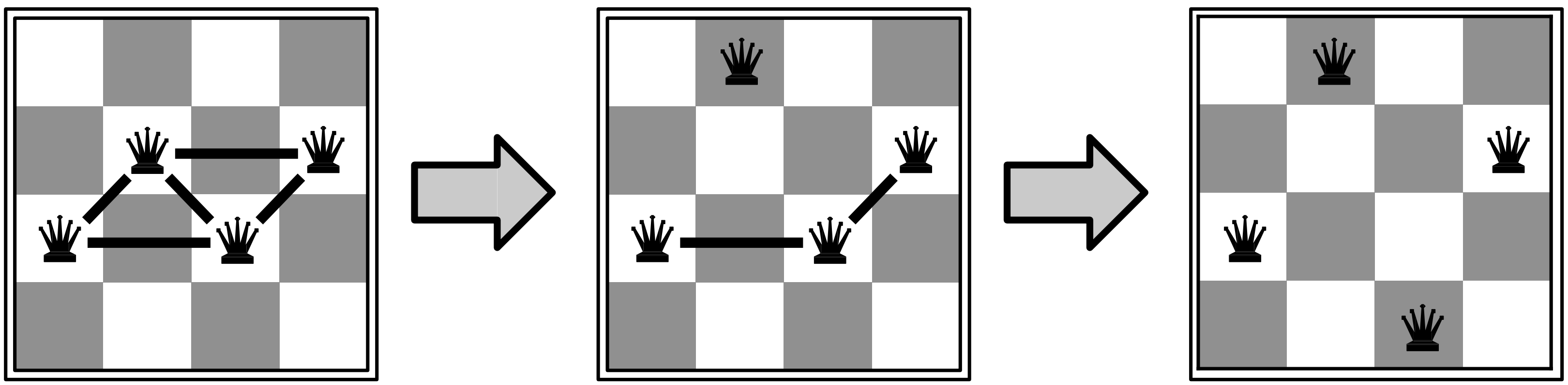
Variants of this approach get within 1% of optimal very quickly with thousands of cities

(Optimum solution is NP-hard. This is not optimum...but close enough?

Example: *N*-queens Problem

Start: Put n queens on an $n \times n$ board with no two queens on the same row, column, or diagonal

Plan: Move a single queen to reduce number of conflicts → generates next board



h = 5 h = 2 h = 0

Almost always solves n-queens problems almost instantaneously for very large n, e.g., n = 1 *million*

(Ponder: how long does N-Queens take with DFS?)

Hill-climbing Search

Plan: From current state, always move to adjacent state with highest value

- "Value" of state: provided by *objective function*
  - *Essentially identical to goal heuristic h(n) from Ch.3*

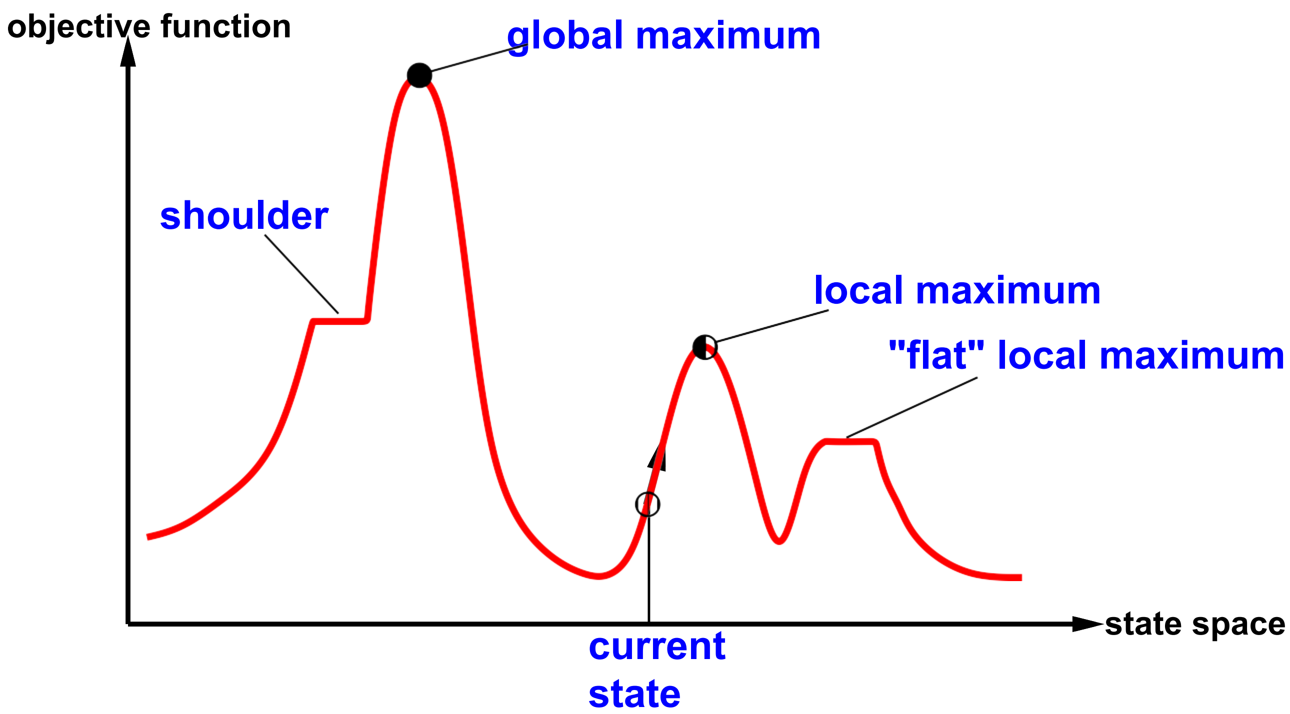
- Always have just one state in memory!

"Like climbing Everest ... in thick fog ... with amnesia"

```
function Hill-Climbing( problem) returns a state that is a local maximum
   inputs: problem, a problem
   local variables: current, a node
                    neighbor, a node
   current ← Make-Node(Initial-State[problem])
   loop do
       neighbor ← a highest-valued successor of current
       if Value[neighbor] ≤ Value[current] then return State[current]
       current ← neighbor
   end
```

Hill-climbing: challenges

Useful to consider state space landscape



"Greedy" nature → can get stuck in:

- Local maxima
- Ridges: ascending series but with downhill steps in between
- Plateau: shoulder or flat area.

Hill climbing: Getting unstuck

Pure hill climbing search on 8-queens: gets stuck 86% of time! 14% success

Overall Observation: "greediness" insists on always uphill moves

Overall Plan for all variants: Build in ways to allow *some* non-optimal moves
→ get out of local maximum and onward to global maximum

Hill climbing modifications and variants:

- Allow sideways moves hoping plateau is shoulder, will find uphill gradient
  - but limit the number of them! (allow 100: 8-queens= 94% success!)
- Stochastic hill-climbing Choose randomly between uphill successors
  - choice weighted by steepness of uphill move
- First-choice: randomly generate successors until find an uphill one
  - not necessarily the most uphill one → so essentially stochastic too.
- Random restart: do successive hill-climbing searches
  - start at random start state each time
  - guaranteed to find a goal eventually
  - the most you do, the more chance of optimizing goal

Simulated annealing

Based metaphorically on metalic annealing

Idea:

- ✓ escape local maxima by allowing some random "bad" moves
- ✓ but gradually decrease the degree and frequency
- ✓ → jiggle hard at beginning, then less and less to find global maxima

```
function Simulated-Annealing( problem, schedule) returns a solution state
    inputs: problem, a problem
            schedule, a mapping from time to "temperature"
    local variables: current, a node
                     next, a node
                     T, a "temperature" controlling prob. of downward steps

    current ← Make-Node(Initial-State[problem])
    for t ← 1 to ∞ do
        T ← schedule[t]
        if T = 0 then return current
        next ← a randomly selected successor of current
        ΔE ← Value[next] – Value[current]
        if ΔE > 0 then current ← next
        else current ← next only with probability e^(ΔE/T)
```

Properties of Simulated Annealing

- Theoretical guarantee:
  - Stationary distribution: $p(x) \propto e^{\frac{E(x)}{kT}}$
  - If T decreased slowly enough, will converge to optimal state!
- Is this an interesting guarantee?
- Sounds like magic, but reality is reality:
  - The more downhill steps you need to escape, the less likely you are to every make them all in a row
  - People think hard about *ridge operators* which let you jump around the space in better ways

- Widely used in VLSI layout, airline scheduling, etc.

Local beam search

Observation: we do have *some* memory. Why not use it?

Plan: keep k states instead of 1

- choose top k of *all their* successors
- Not the same as k searches run in parallel!
- Searches that find good states place more successors in top k
  → "recruit" other searches to join them

Problem: quite often, all k states end up on same local maximum

Solution: add stochastic element

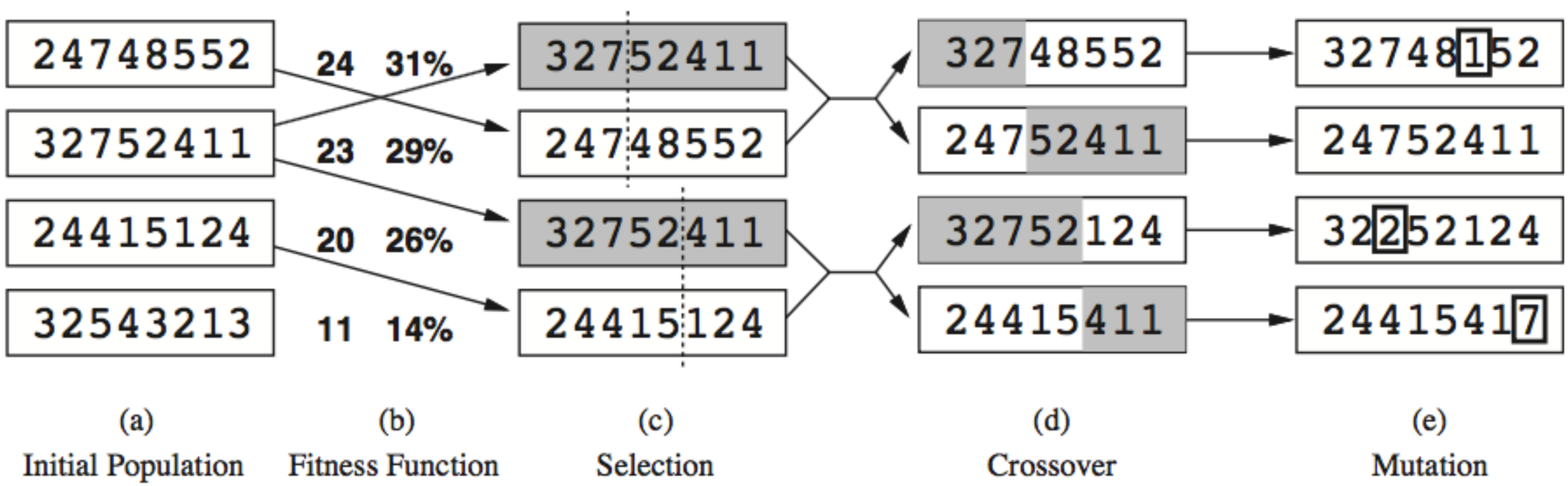
- choose k successors randomly, biased towards good ones
- note: a fairly close analogy to natural selection (survival of fittest)

Genetic algorithms

Metaphor: "breed a better solution"

- Take the best characteristics of two parents → generate offspring

Effectively: stochastic local beam search + generate successors from pairs of states



Steps:

1. Rank current population (of states) by fitness function
2. Select states to cross. Random plus weighted by fitness (more fit=more likely)
3. Randomly select "crossover point"
4. Swap out whole parts of states to generate "offspring"
5. Throw in mutation step (randomness!)

Genetic Algorithm: N-Queens example

- Why does crossover make sense here?
- When wouldn't it make sense?
- What would mutation be?
- What would a good fitness function be?

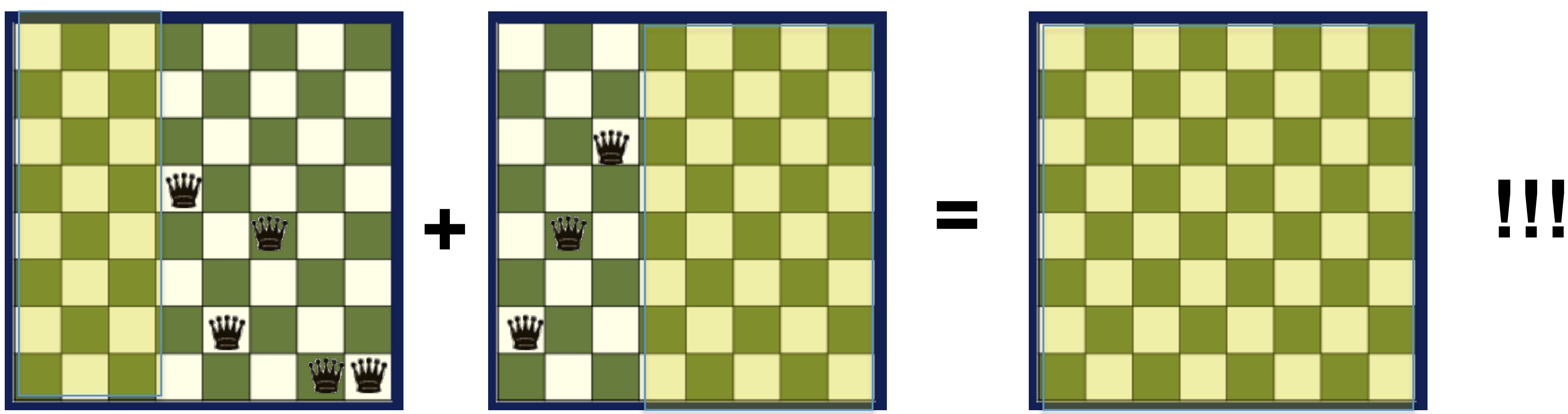
Genetic algorithms: analysis

Pro: Can jump search around the search space...

- In larger jumps. Successors not just one move away from parents
- In "directed randomness". Hopefully directed towards "best traits"
- In theory: find goals (or optimum solutions) faster, more likely.

Concerns: Only really works in "certain" situations...

- States must be encodable as strings (to allow swapping pieces)
- Only really works if substrings somehow related functionally meaningful pieces.
  → counter-example:



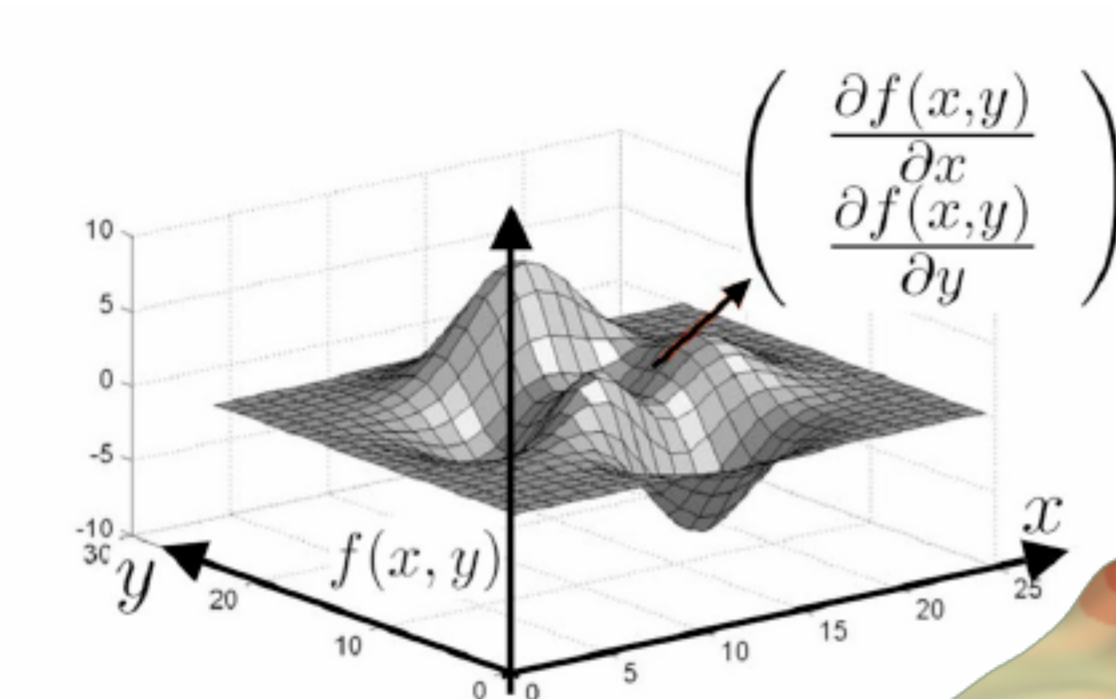
Overall: Genetic algorithms are a cool, but quite specialized technique

- Depend *heavily* on careful engineering of state representation
- Much work being done to characterize promising conditions for use.

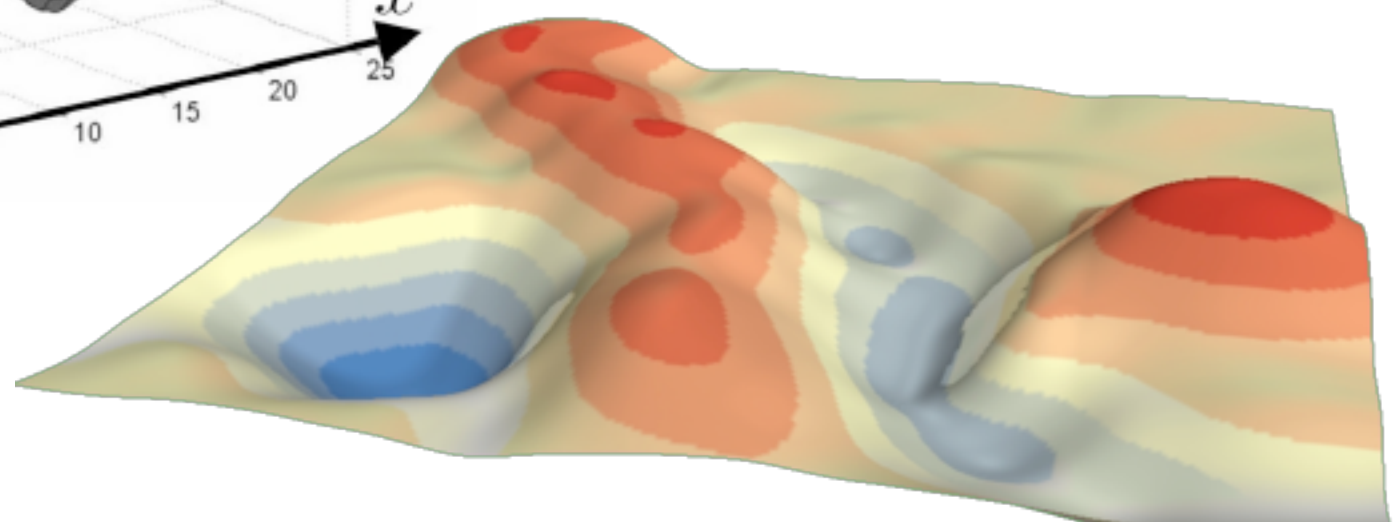
Searching in continuous state spaces (briefly…)

Observation: so far, states have been discrete "moves" apart

- Each "move" corresponds to an "atomic action" (can't do a half-action! 1/16 action
- But the real world is generally a continuous space!
- What if we want to plan in real world space, rather than logical space?



From researchGate.net



Katieluethgeospatial.blogspot.com

Searching Continuous spaces

Example: Suppose we want to site three airports in Romania:

- 6-D state space defined by (x_1, y_2), (x_2, y_2), (x_3, y_3)
- objective function $f(x_1, y_2, x_2, y_2, x_3, y_3)$ = sum of squared distances from each city to nearest airport (six dimensional search space)

Approaches:

Discretization methods turn continuous space into discrete space

- e.g., empirical gradient search considers $\pm\delta$ change in each coordinate
- If you make δ small enough, you get needed accuracy

Gradient methods actually *compute* a gradient vector as a continuous fn.

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial y_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial y_2}, \frac{\partial f}{\partial x_3}, \frac{\partial f}{\partial y_3}\right)$$

to increase/reduce f, e.g., by $\mathbf{x} \leftarrow \mathbf{x} + \alpha \nabla f(\mathbf{x})$

Summary: interesting area, highly complex

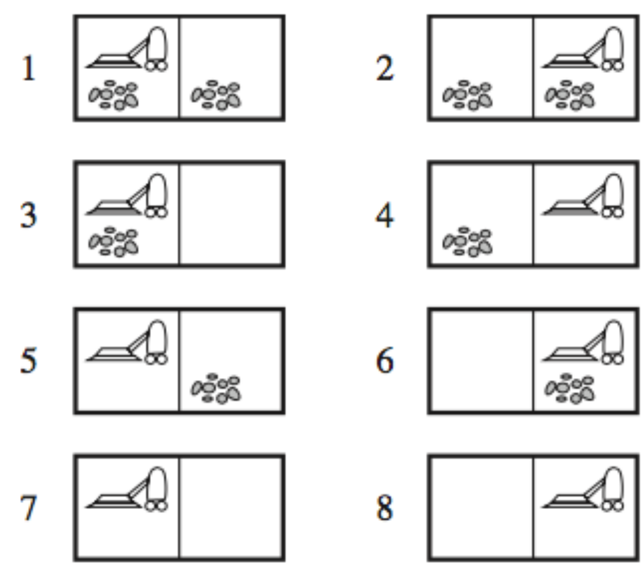
Searching with Non-deterministic actions

- So far: fully-observable, deterministic worlds.
  - Agent knows exact state. All actions *always* produce *one* outcome.
  - Unrealistic?

- Real world = partially observable, non-deterministic
  - Percepts become useful: can tell agent *which* action occurred
  - Goal: not a simple action sequence, but *contingency plan*

- Example: Vacuum world, v2.0
  - Suck(p1, dirty)= (p1,clean)
    and *sometimes* (p2, clean)
  - Suck(p1, clean)= *sometimes* (p1,dirty)

  - If start state=1, solution=
    [Suck, if(state=5) then [right,suck]]

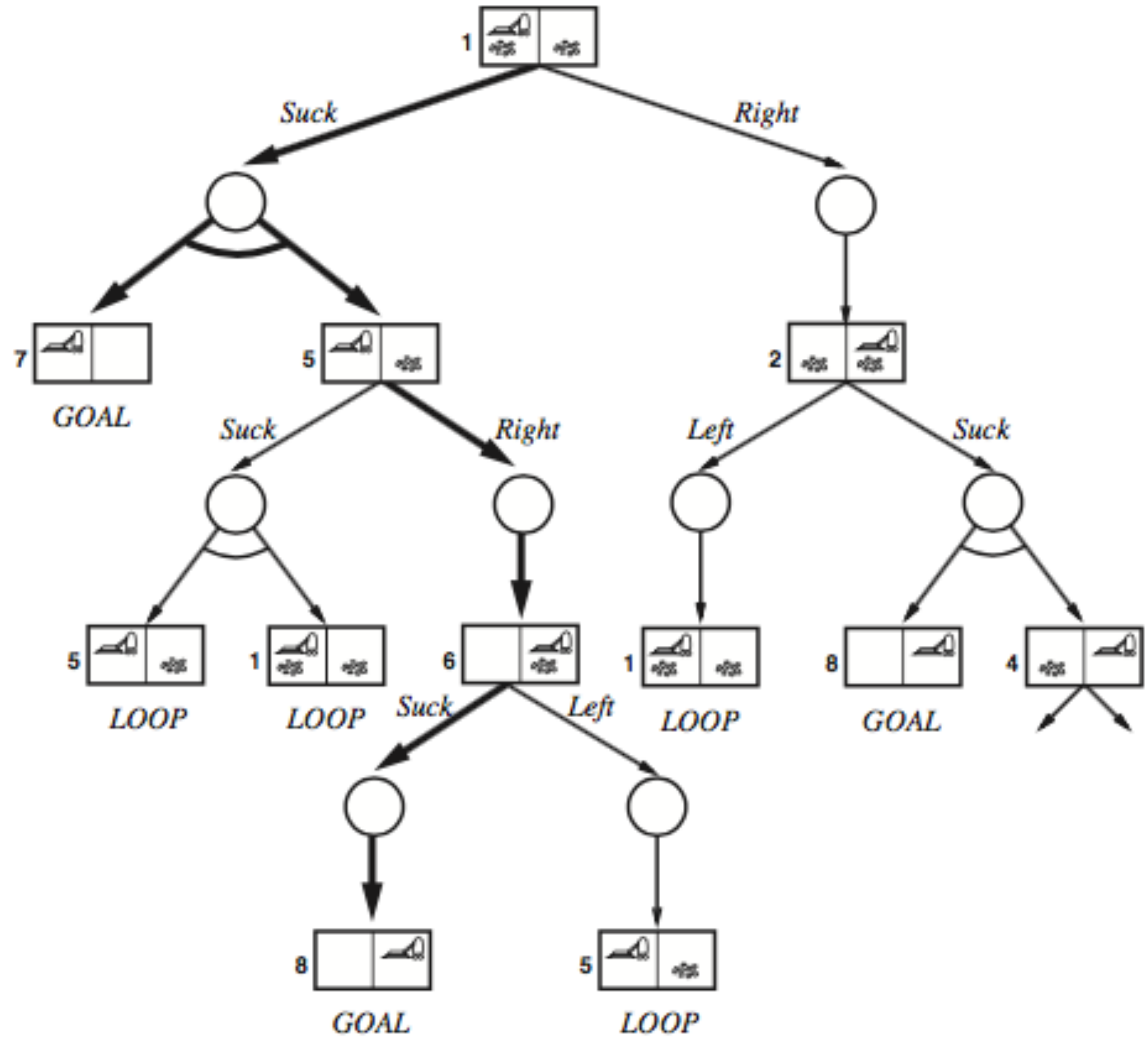


AND-OR trees to represent non-determinism

- Need a different kind of search tree
  - When search agent chooses an action: OR node
    - Agent can specifically choose one action *or* another to include in plan.
    - In Ch3 : trees with only OR nodes.
  - Non-deterministic action= there may be *several* possible outcomes
    - Plan being developed must cover *all possible outcomes*
    - AND node: because must plan down all branches too.
- Search space is an AND-OR tree
  - Alternating OR and AND layers
  - Find solution= search this tree using same methods from Ch3.
- Solution in a non-deterministic search space
  - Not simple action sequence
  - Solution= *subtree* within search tree with:
    - Goal node at each leaf (plan covers all contingencies)
    - One action at each OR node
    - A branch at AND nodes, representing all possible outcomes
- Execution of a solution = essentially “action, case-stmt, action, case-sttmt”.

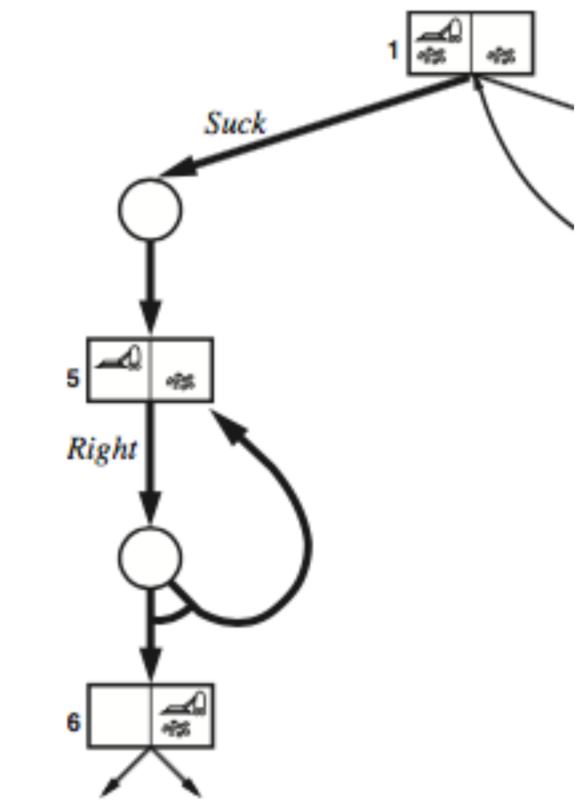
Non-deterministic search trees

- Start state = 1
- One solution:
  1. Suck,
  2. if(state=5) then [right,suck]]
- What about the “loop” leaves?
  - Dead end?
  - Discarded?



Non-determinism: Actions that fail

- Action *failure* is often a non-deterministic outcome
  - Creates a cycle in the search tree
- If no successful solution (plan) without a cycle:
  - May return a solution that *contains* a cycle
  - Represents *retrying* the action
- Infinite loop in plan execution?
  - Depends on environment
    - Action guaranteed to succeed eventually?
  - In practice: can limit loops
    - Plan no longer complete (could fail)



Searching with Partial Observations

- Previously: Percept gives full picture of state
  - eg. Whole chess board, whole boggle board, entire robot maze
- Partial Observation: incomplete glimpse of current state
  - Agent's percept: zero <= percept < full state
  - Consequence: we don't always know exactly what state we're in.
- Concept of *believe state*
  - set of *all possible* states agent *could* be in.
- Find a solution (action sequence) that the leads to goal
  - Actions applied to a believe state → new believe state based on *union* of that action applied to all real states within believe state

Conformant (sensorless) search

- Worst possible case: percept= null. Blind!
  - Actually quite useful: finds plan that works regardless of sensor failure

- Plan:
  - Build a belief state space based on the real state space
  - Search that state space using the usual search techniques!

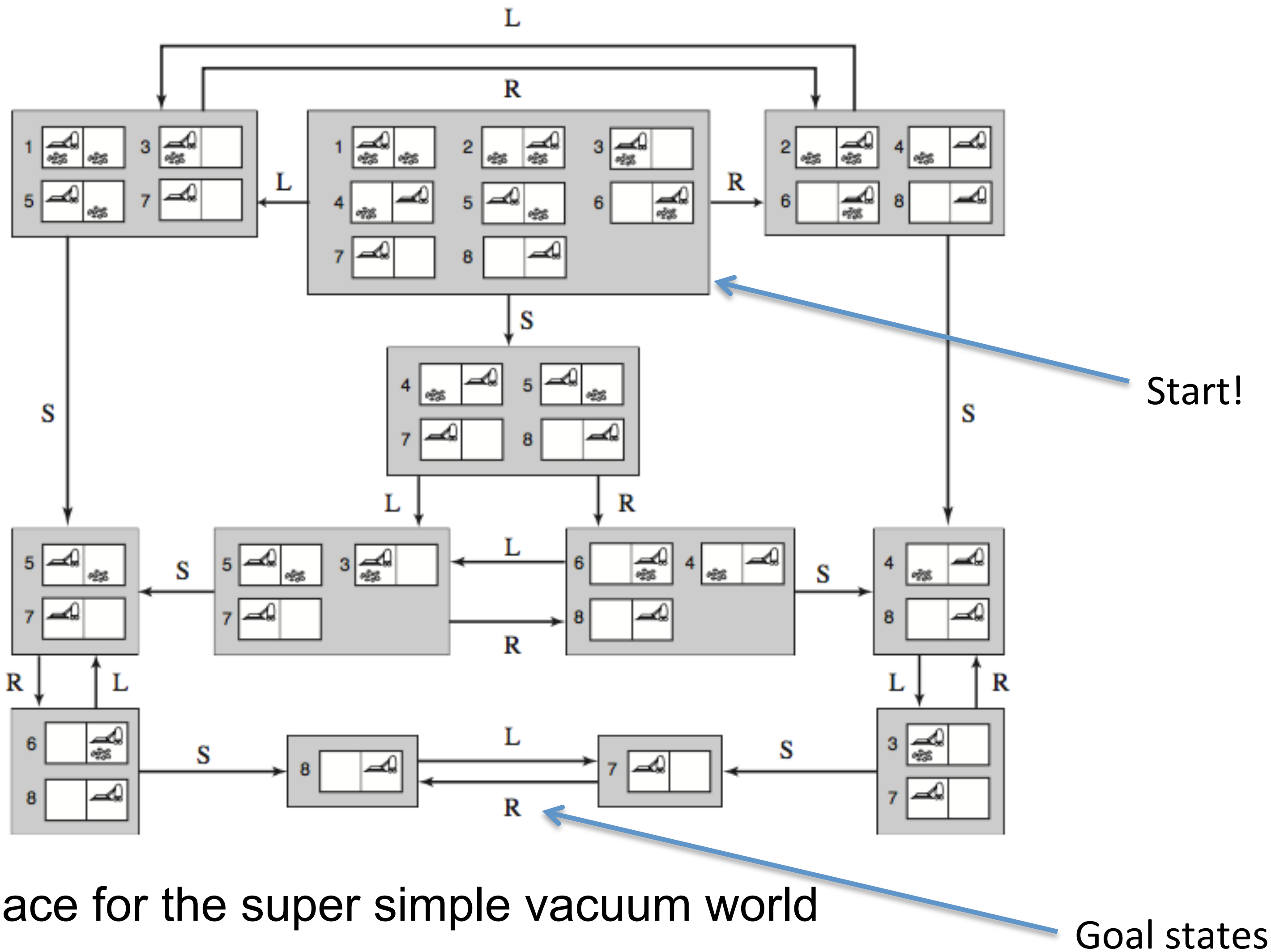
- Belief state space:
  - Believe states: Power-set(real states).
    - Huge! All possible combinations! N physical states = 2^N believe states!
    - Usually: only small subset actually reachable!

  - Initial State: All states in world
    - No sensor input = no idea what state I'm really in.
    - So I "believe" I might be in any of them.

Conformant (sensorless) search

- Belief state space (cont.):
  - Actions: basically same actions as in physical space.
    - For simplicity: Assume that illegal actions have no effect
    - Example: Move(left, p1) = p1 if p1 is the left edge of the board.
    - Can adapt for contexts in which illegal actions are fatal (more complex).
  - Transitions (applying actions):
    - Essentially take Union of action applied to all physical states in belief state
    - Example: b={s1,s2,s3), then action(b) = Union(action(s1), action(s2),action(s3))
    - If non-deterministic actions: just Union *the set of states* that each action produces.
  - Goal Test: Plan must work regardless!
    - Believe state is goal *iff* all physical states it contains are goals!
  - Path cost: tricky
    - What if a given action has different costs of different physical states?
    - Assume for now: all actions = same cost in all physical states.
- With this framework:
  - can *automatically* construct belief space from any physical space
  - Now simply search belief space using standard algos.

Conformant (sensorless) search: Example space

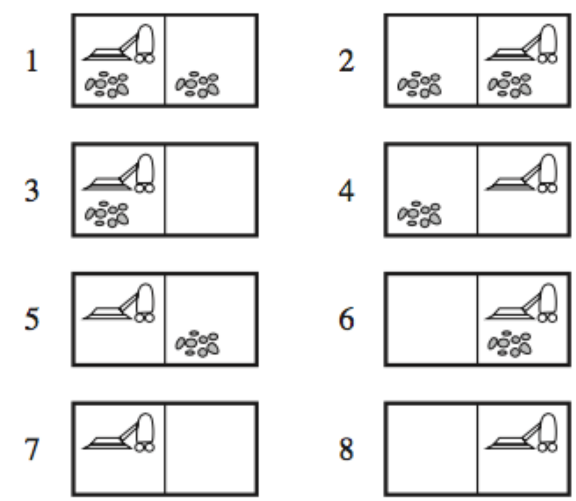


- Belief state space for the super simple vacuum world
- Observations:
  - Only 12 reachable states. Versus 2^8= 256 possible belief states
  - State space still gets huge very fast! → seldom feasible in practice
  - We need sensors! → Reduce state space greatly!

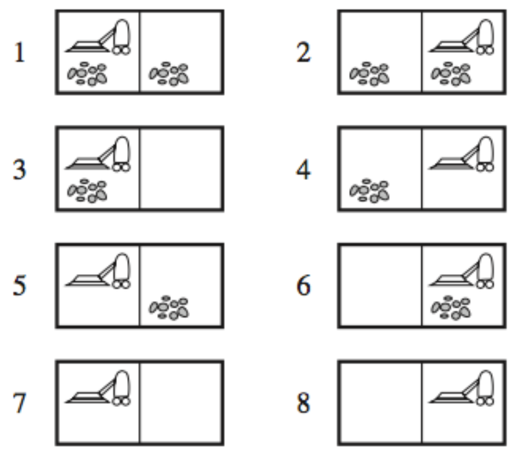
Searching with Observations (percepts)

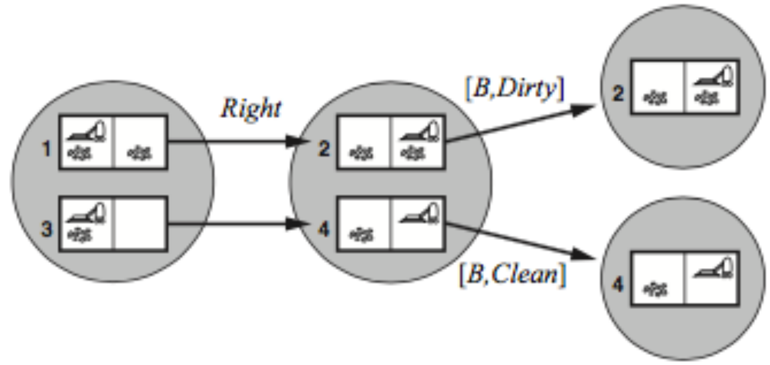
- Obviously: must state what percepts are available
  - Specify what part of "state" is observable at each percept
  - Ex: Vacuum knows position in room, plus if local square dirty
    - But no info about rest of squares/space.
    - In state 1, Percept = [A, dirty]
    - If sensing non-deterministic → could return *a set* of possible percepts → multiple possible belief states

- So now transitions are:
  - Predict: apply *action* to each physical states in belief state to get new belief state
    - Like sensorless
  - Observe: gather percept
    - Or percept**s**, if non-det.
  - Update: filter belief state based on percepts



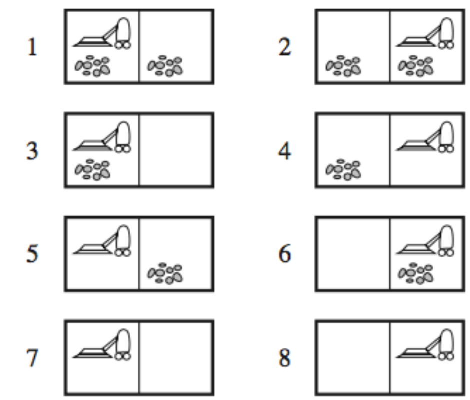
Example: partial percepts





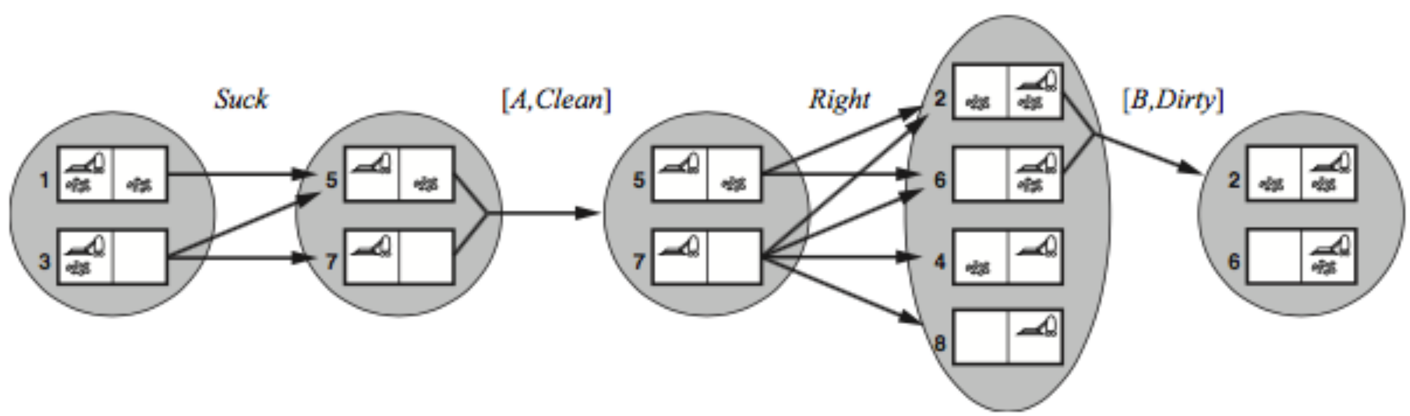
- Initial percept = [A, dirty]
- Partial observation = partial certainty
  - Percept could have been produced by *several* states (1...or 3)
  - Predict: Apply Action → new belief state
  - Observe: Consider possible percepts in new b-state
  - Update: New percepts then *prune* belief space
    - Percepts (may) rule out some physical states in the belief state.
    - Generates successor options in tree
  - Look! Updated belief states *no larger than* parents!!
    - Observations *can only help* reduce uncertainty → much better than sensorless state space explosion!

Searching/acting in partially observable worlds



- Searching for goal = find viable plan
  - Use same standard search techniques
    - Nodes, actions, successors
    - Dynamically generate AND-OR tree
    - Goal = subtree where all leaves are goal states
  - Just like sensorless...but pruned by percepts!

- Action! An agent to execute the plan you find
  - Execute the conditional plan that was produced
    - Branches at each place where multiple percepts possible.
    - Agent tests its *actual* percept at branch points → follows branch
    - Maintains its current belief state as it goes



Online Search

- So far: Considered “offline” search problem
  - Works “offline” → searches to compute a whole plan...*before ever acting*
  - Even with percepts → gets HUGE fast in real world
    - Lots of possible actions, lots of possible percepts...plus non-det.

- Online search
  - Idea: Search as you go. Interleave search + action
  - Pro: *actual* percepts prune huge subtrees of search space @ each move
  - Con: plan ahead less → don’t foresee problems
    - Best case = wasted effort. Reverse actions and re-plan
    - Worst case: not reversible actions. Stuck!

- Online search only possible method in some worlds
  - Agent doesn’t know what states exist (exploration problem)
  - Agent doesn’t know what effect actions have (discovery learning)
  - Possibly: do online search for *awhile*
    - until learn enough to do more predictive search

The nature of active online search

- Executing online search = algorithm for planning/acting
  - *Very different* than offline search algos!
  - Offline: search virtually for a plan in constructed search space...
    - Can use any search algorithm, e.g., A* with strong h(n)
    - A* can expand any node it wants on the frontier (jump around)
  - Online agent: Agent literally *is in some place*!
    - Agent *is at* one node (state) on frontier of search tree
    - Can't just jump around to other states...must plan from current state.
    - (Modified) Depth first algorithms are ideal candidates!
  - Heuristic functions remain critical!
    - H(n) tells depth first *which* of the successors to explore!
    - Admissibility remains relevant too: want to explore *likely* optimal paths first
    - Real agent = real results. At some point I find the goal
      - Can compare actual path cost to that predicted at each state by H(n)
      - **Competitive Ratio:** Actual path cost/predicted cost. Lower is better.
      - Could also be basis for developing (learning!) improved H(n) over time.

Online *Local* Search for Agents

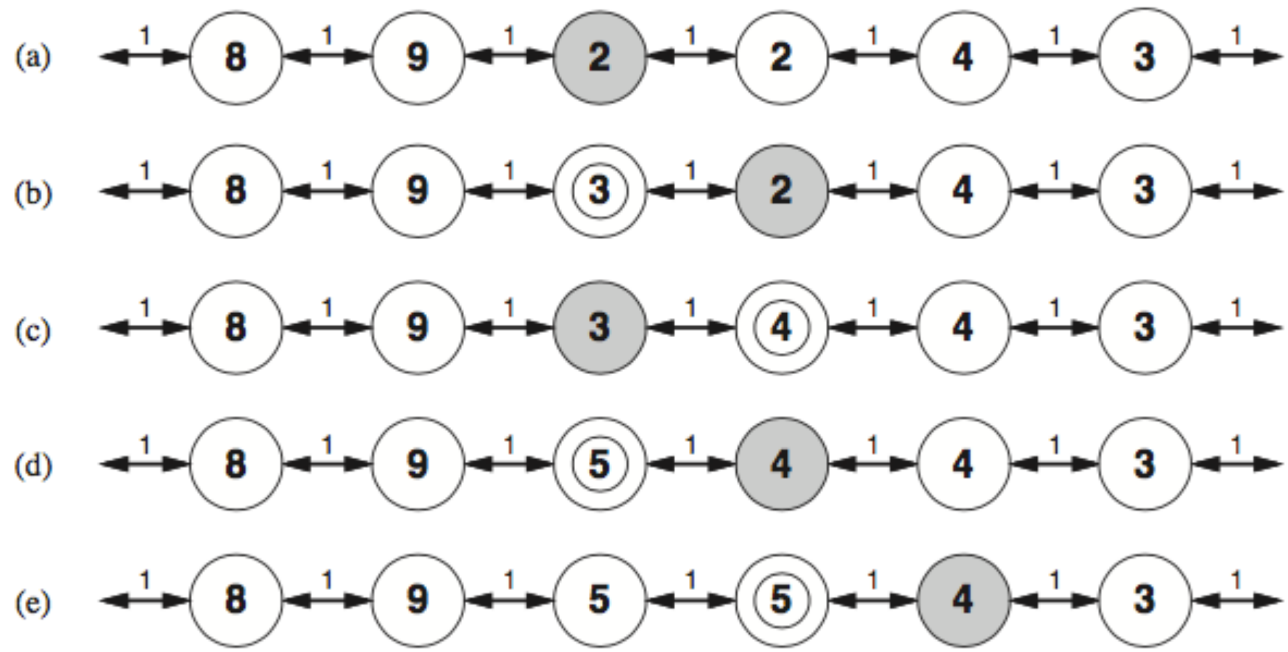
- What if search space is very bushy?
  - Even IDS version of depth-first are too costly
  - Tight time constraints could also limit search time
- Can use our other tool for local search!
  - Hill-climbing (and variants)

- Problem: agents in *in the physical world, operating*
  - Random restart methods for avoiding local minima are problematic
    - Can’t just move robot back to start all the time!
  - Random Walk approaches (highly stochastic hill-climbing) can work
  - Will *eventually* wander across the goal place/state.

- Random walk + *memory* can be helpful
  - Chooses random moves but…
  - remembers where it’s been, and updates costs along the way
  - Effect: can “rock” its way out of local minima to continue search

Online *Local* Search for Agents

- Result: Learning Real-time A* (LRTA*)



- Idea: memory = *update* the h(n) for nodes you've visited
  - When stuck use: h(n) = cost(n → best neighbor) + h(neighbor)
  - Update the h(n) to reflect this. If you ever go back there, h(n) is higher
  - You "fill in" the local minimum as you cycle a few times. Then escape...

- LRTA* → many variants; vary in selecting next action and updating rules

Chapter 4: Summary

- Search techniques from Ch.3
  - still form basic foundation for possible search variants
  - Are not well-suited *directly* to many real-world problems
    - Pure size and bushiness of search spaces
    - Non-determinism. In Action outcomes. In Sensor reliability.
    - Partial observability. Can see *all* features of current state.

- Classic search must be adapted and modified for the real world
  - Hill-climbing: can be seen as DFS + h(n) ... with depth limit of **one.**
  - Beam search: can be seen as Best First...with Frontier queue limit = k.
  - Stochastic techniques (incl. simulated annealing) = seen as Best-first with weighted randomized Q selection.
  - Belief State Search = identical to normal search...only searching belief space
  - Online Search: Applied DFS or local searching
    - With high cost of backtracking and becoming stuck
    - Pruning by moving before complete plans made.


STOP