A Simulated Annealing Algorithm for Optimizing RF Power Efficiency in Coupled-Cavity Traveling-Wave Tubes

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***Abstract*—Decreasing the radio frequency (RF) phase velocity in the output section of a traveling-wave tube (TWT) is a technique commonly used to increase RF power efficiency. In order to optimize the profile of the phase velocity, a simulated annealing algorithm has been developed and implemented into the NASA multidimensional large-signal coupled-cavity TWT computer model. This algorithm allows the determination of the lengths of the individual cavities at the end of the output section necessary to provide the optimized phase velocity profile. The resulting nonlinear computer-generated phase velocity profile provides a design with optimized RF efficiency. In this paper, the optimization algorithm will be described and computational results will be shown. These results indicate an increase in center-frequency RF efficiency from 7.1 to 13.5% for a V-band coupled-cavity TWT.**

I. Introduction

THE mechanism for RF power amplification in a traveling-wave tube (TWT) involves the transfer of kinetic energy from electrons in an electron beam to the electromagnetic field of the traveling radio-frequency (RF) wave. In order to maximize this energy transfer, the majority of electrons in the beam must travel at near synchronous velocities with the phase velocity of the RF wave. This is generally accomplished by decreasing the circuit period in the last part of the TWT circuit, so that the phase velocity decreases in step with the decelerating electrons in what is referred to as a phase velocity taper.

Early phase velocity tapers for coupled-cavity TWT's used linearly decreasing period tapers with up to three constant period sections [1]–[3] to increase RF power efficiency. More recently, the phase-adjusted taper (PAT) design procedure [4] was developed. A PAT was used to more than double the peak RF efficiency of a ferruleless coupled-cavity TWT [5]. Although successful, the PAT design methodology was limited to producing phase velocity profiles that maintained a constant relationship between the circuit phase velocity and the electron bunch velocity in the taper.

This paper describes an improved and more generalized approach to TWT phase velocity taper design optimization based on a simulated annealing algorithm. The major advantage of simulated annealing is that it enables a globally optimized solution to be determined, while other optimization techniques converge on the closest local extremum. The algorithm is described and computational results from its implementation into the NASA multidimensional large-signal coupled-cavity TWT computer model are presented.

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II. Background

In conventional optimization procedures, perturbations are made to the input design variables. If the perturbations improve the output parameter to be optimized, more perturbations are made in the same direction. This continues until the output parameter no longer improves. These procedures can cause the solution to be easily trapped in a local extremum, that is, one that is optimal with respect to a small neighborhood of input variables, but is not necessarily the global optimum over the entire range of possible input variable values. Simulated annealing avoids this problem by carefully allowing the configuration of input variables to temporarily make the output parameter worse, enabling the solution to jump out of a local extremum and fall into a more productive path toward the global optimum.

The concept of combinatorial optimization by simulated annealing was introduced in the early 1980's [6]. The idea is derived from the annealing process in condensed matter physics, a thermal procedure for obtaining the lowest energy (ground) state for a solid. In this procedure, the solid is first heated to just below its melting temperature and is then slowly cooled. This causes the randomly arranged atoms to gradually arrange into the highly structured lattice of the ground state in which the energy of the system is minimized. The essence of the procedure is that the slow transformation from a random to a highly ordered state allows the atoms to escape from meta-stable energy states with localized energy minima. If the initial temperature is not high enough or if the cooling is too rapid, the resulting solid will be at a higher energy than the ground state.

The simulated annealing formulation is based on the Metropolis algorithm [7]. From the starting configuration of the heated solid, a random atom is moved to a new location. If the resulting energy is reduced, the new configuration is automatically accepted as the starting point for the next move. However, if the energy increases, the new configuration also has a possibility of being accepted. This possibility is high at high temperatures and decreases as the temperature cools.

The probability of acceptance $P(\Delta E)$ is modeled with a Boltzmann distribution

$$P(\Delta E) = e^{-\Delta E/kT} \tag{1}$$

where ΔE is the change in energy and kT is a base energy with k as Boltzmann's constant and T as temperature. This probability is then compared to a random number between 0 and 1. If the random number selected is less than $P(\Delta E)$, the new configuration is saved; if not, the previous configuration is used to start the next step. This basic step is repeated many times until equilibrium is reached at temperature T. This step continues to be repeated at a succession of lower temperatures until final equilibrium is attained. The beginning temperature is high enough to allow most uphill moves. As the temperature cools, fewer uphill moves are allowed, until at the coldest temperature the solution freezes into its final form with virtually no chance of an uphill move being accepted. In applying simulated annealing to general problems, the energy in the Metropolis algorithm can be replaced by any quantity to be optimized. Simulated annealing has been used to solve optimization problems in a wide variety of areas including operations research, very large-scale integrated (VLSI) circuit design, code design, image processing, and molecular physics [8].

III. Analysis

The NASA multidimensional large-signal coupled-cavity TWT model [9]–[12] determines the interaction between a two dimensional slow-wave RF circuit field and an electron beam with disks or rings of electrical charge which have axial, radial, and azimuthal velocity components. The amplitude and phase of the RF circuit electromagnetic field and the trajectories of the electron disks or rings are determined from the calculated axial and radial space-charge, circuit and magnetic forces as the disks or rings pass through the individual cavities of the circuit. Independent geometrical and electrical parameters are input for each cavity.

The Hughes Aircraft Company 961HA TWT, a 59–64 GHz, 75-W coupled-cavity TWT developed under NASA Contract no. NAS3-25 090 [13], was used as a baseline for this study. Previously, this TWT was simulated with the NASA model and very good agreement with experimental results for the output RF power from 60 to 64 GHz [14] was achieved. The end of the output section has a mild two-step phase velocity taper (Fig. 1). This taper was designed to provide 75 W of RF power over the 5-GHz bandwidth. In this study, the taper design will be optimized for RF power efficiency at the center frequency of 61.5 GHz, without consideration of the performance over the bandwidth range. Further work will be required to optimize the design over a wide bandwidth.

The following simulated annealing algorithm was developed and implemented into the NASA model to determine an optimized phase velocity taper:

Step 1: The initial conditions for the taper were selected. These included the starting and ending cavities of the taper and the initial length of each cavity.

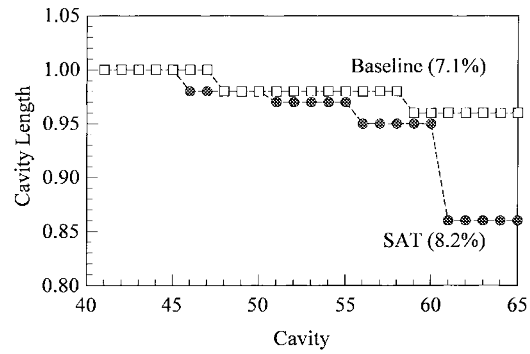

Fig. 1. The cavity length profiles for the phase velocity tapers at the end of the output sections for the baseline Hughes 961HA TWT and a simulated annealing taper (SAT). The baseline TWT, which was designed for wide-bandwidth performance, has a center-frequency (61.5 GHz) RF efficiency of 7.1%; while the SAT has a computed efficiency of 8.2%.

Step 2: A typical taper for this TWT is on the order of 20–40 cavities long. To optimize the length of each of these cavities would require extensive computational time with the processor that is currently available. Therefore, the taper was divided into segments of five equal-length cavities each. At each pass, a segment was selected by a random number generator. The first time that a segment was selected, the length of each cavity in the segment was increased by 1%. On succeeding alternate selections of a segment, the lengths were decreased and increased by 1%. With each change in cavity length, the impedance, phase shift per cavity, and gap length were adjusted. From calculations with the three-dimensional electromagnetic simulation code MAFIA [14], [15], it is a very good approximation for this TWT at the center frequency of 61.5 GHz to assume that the impedance decreases 1.97% and that the phase shift per cavity decreases 0.179% for every 1% decrease in cavity length. The gap/cavity length ratio was kept constant. The attenuation per cavity has a weak dependence on length and was assumed to be constant.

Step 3: With the new taper cavity parameters, the NASA multidimensional large-signal coupled-cavity TWT model was used to evaluate the new RF power efficiency.

Step 4: With a random number generator, a value R was obtained between 0 and 1. This value was compared to the probability of acceptance defined as

$$P = e^{(\eta' - \eta)/T} \tag{2}$$

where η' and η were the new and old values of efficiency, respectively. The control parameter, T, is a dimensionless quantity that corresponds to temperature in the physical analogy. If $R < P$, the new cavity length of Step 2 was accepted; if $R > P$, the new length was rejected. Note that if the efficiency increases, acceptance of the new length was insured. If the efficiency decreased, acceptance could occur but became less probable as T decreased.

Step 5: Steps 2–4 were repeated for a total of N passes. If there was at least one acceptance after N passes, the value of T was decreased by a reduction factor designated by r_T and Steps 2–5 were repeated. Convergence to a final solution was assumed after there were no acceptances with N passes at a value of T.

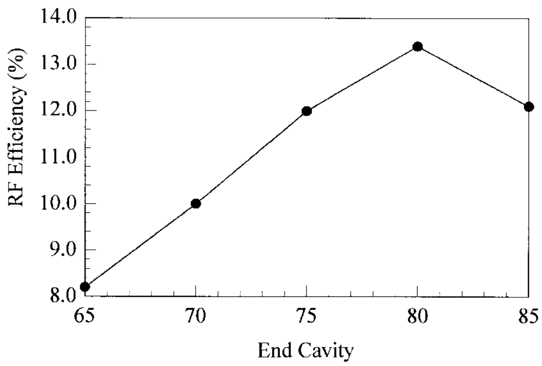

Fig. 2. The computed RF efficiencies for SAT's as a function of length. The maximum occurs with an end cavity of 80, indicating an output circuit 15 cavities longer than that of the baseline TWT.

IV. Results and Discussion

The phase velocity tapers calculated with the simulated annealing algorithm described above will be compared to that of the baseline TWT. The output section of this TWT contains 65 cavities, of which 47 are a standard length of 0.096 77 cm, 11 are reduced in length by 2%, and 7 are reduced 4%. The cavity lengths at the end of the output section are shown in Fig. 1.

Before beginning the simulated annealing algorithm, the "cooling schedule" must be selected. This is defined by the values for the following parameters which were described in the previous section: r_T, N, and the initial value of $T(T_0)$. In order for a globally optimized solution to be obtained, T_0 should be large enough so that at the beginning of the optimization a decided majority of the proposed cavity length changes is accepted. It is also necessary that T decrease sufficiently slowly.

The algorithm was used with a variety of cooling schedule parameters to calculate simulated annealing tapers (SAT's) from cavity 46 to 65, starting with all cavity lengths equal to that of a standard cavity. The parameter values which provided a sufficient cooling schedule to obtain an optimized solution were $T_0 = 0.002, r_T = 0.5$, and $N = 10n$, where n is the number of cavity segments in the taper. The resulting SAT, as shown in Fig. 1, was obtained after 270 passes. Although it is a much stronger taper than that of the baseline TWT, it only increased the RF power efficiency from 7.1 to 8.2%. However, by applying the algorithm to longer output sections, significantly higher values of efficiency were obtained. Fig. 2 shows the resulting efficiencies for SAT's as a function of end cavity. A clear maximum of 13.4% occurs with an end cavity of 80.

The cooling schedule was reexamined with SAT's from cavity 46 to 80 and it was found that increasing r_T from 0.5 to 0.7 resulted in an SAT with a slightly higher efficiency of 13.5%, which is shown in Fig. 3. (Further increases in r_T and N did not increase the efficiency.) The progressions of efficiency and accept rate for this taper are shown in Fig. 4. The efficiency increases very rapidly during the first 200 passes while the majority of the proposed length changes are being accepted. As the taper profile converges to its final form, the efficiency increase slows down and the accept rate drops rapidly, reaching zero after 770 passes.

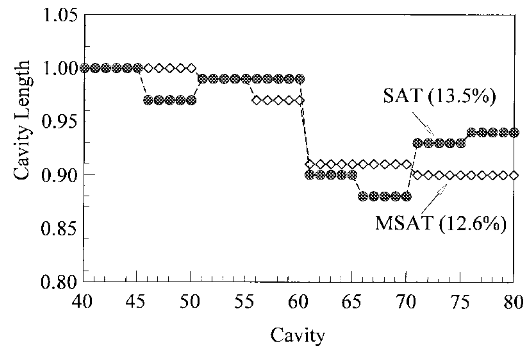

Fig. 3. The best SAT is obtained with an output circuit length of 80 cavities and has a computed RF efficiency of 13.5%. For comparison, a monotonic SAT (MSAT) was determined, in which the generating algorithm was modified to prevent a cavity from being longer than the previous cavity. The MSAT achieves an RF efficiency of only 12.6%.

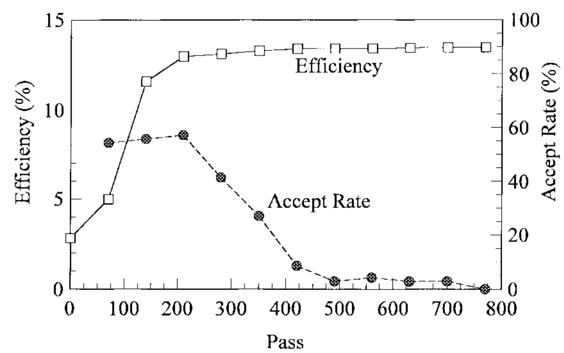

Fig. 4. The progressions of RF efficiency and algorithm accept rate for the determination of the SAT of Fig. 3.

To test the validity of dividing the taper into five-cavity segments, two very computationally intensive runs, which allowed each cavity in the taper to vary, were submitted. The cooling schedule parameters for the first run were the same as for the SAT of Fig. 3. In the second run, the value of N was increased from 10 to 15. Neither run resulting in an improvement in efficiency, suggesting that dividing the taper into five-cavity segments does not sacrifice the design efficiency.

The profile of the SAT in Fig. 3, is interesting in that instead of decreasing smoothly and monotonically, it decreases sharply at cavity 60 and then reverses to increase cavity lengths and RF phase velocity in the last part of the taper. In order to understand this behavior, a comparison taper was determined with the algorithm modified to force a monotonic decrease in cavity length. The resulting monotonic simulated annealing taper (MSAT) is also shown in Fig. 3, and achieves an RF power efficiency of only 12.6%. The kinetic energies of the electronic disks in the SAT and MSAT were examined for each taper at every five cavities. The 24 electron disks of the model were divided into two groups: the fast disks and the slow disks, with kinetic energies greater and lesser than the median, respectively. The average normalized kinetic energies for each group are shown in Fig. 5. For both tapers, the kinetic energy of the disk groups decrease as the electrons give up energy to the RF signal. Also for both tapers, the spread between the fast and slow disks increase as the electron bunch deteriorates. The energies are quite similar for the two tapers except at

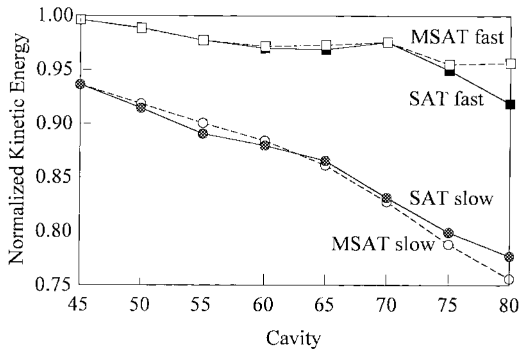

Fig. 5. The average normalized kinetic energies of the slow and fast electron disks of the modeled electron beam for both the SAT and MSAT of Fig. 3. This indicates that the SAT is more effective in decelerating the more energetic electrons at the end of the taper. The kinetic energy is normalized to the initial value at the beginning of the input circuit.

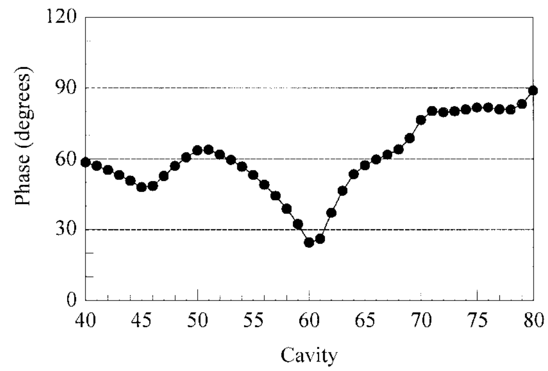

Fig. 6. The phase of the electron bunch with respect to the RF circuit wave for the SAT of Fig. 3. In the region of cavity 60, the small value of phase is conducive to strong bunch formation. The phase rapidly increases toward the domain of strong bunch deceleration (between 80° and 90°) where it remains for an extended length at the end of the taper.

the end. It can be seen that the SAT achieves its superior efficiency by more effectively decreasing the energies of the fast electrons at the end of the taper. (The MSAT is more effective in extracting energy from the slow electrons, but this difference is not as large as that for the fast electrons.) The smaller energy spread between the slow and fast electrons at the end of the circuit for the SAT should also be an advantage in optimizing for overall efficiency, where it is desirable to recover as much energy as possible from the spent electron beam with a multistage-depressed collector.

In order to compare the phase behavior of the SAT with that of phase-adjusted tapers (PAT's) [4], Fig. 6 shows the computed phase of the electron bunch with respect to the RF circuit wave for the SAT of Fig. 3. (The electron bunch phase was determined by calculating the fundamental Fourier component of the axial charge distribution at the gap center of each cavity). In a PAT, the phase of the electron bunch with respect to the RF circuit wave has a value between 0° and 45° in the beginning of the taper in order to enable the formation of a strong bunch [4]. Then throughout the taper, the phase is adjusted linearly upward toward 90° where maximum bunch deceleration and energy extraction occur [4]. As in a PAT, the SAT has a low value of phase in the beginning, with a minimum value of 25° at cavity 60. However, instead of increasing linearly, the phase first increases rapidly from the minimum value to just over 80° at cavity 71 and then levels off over the last part of the taper. It maintains a value between 80° and 90° from cavity 71 through the end cavity 80. Thus it remains in a domain of high bunch deceleration [4] throughout an extended length at the end of the taper, enabling maximum power transfer from the beam to the RF wave.

V. Conclusions

A simulated annealing optimization algorithm has been developed and implemented into the NASA multidimensional large-signal coupled-cavity TWT computer model. The algorithm was tested by using it to determine the profile of cavity lengths at the end of the output section in order to optimize center-frequency RF efficiency for a V-band coupled-cavity TWT. The resulting taper design shows a computed RF efficiency at center frequency of 13.5% compared to 7.1% for the baseline TWT. The taper provides a sharp decrease in RF phase velocity and then a reversal to increase phase velocity at the end. This profile allows the taper to capture more of the energy of the fast electrons than is possible with a monotonic taper. It also allows the phase of the electron bunch with respect to the RF circuit wave to increase rapidly from the domain of strong bunch formation to the domain of strong bunch deceleration. The phase remains in this strong bunch deceleration domain for an extended length at the end of the taper, permitting optimal power transfer from the beam to the RF wave.

A primary advantage of this algorithm is that the simulated annealing allows a global optimum solution to be obtained whereas most optimization algorithms converge on a local optimum. Another major advantage is that the algorithm can be readily adapted to optimize any calculable TWT output characteristic in terms of any combination of the model's cavity, beam, and focusing parameters. Further work is planned to optimize the efficiency over a wide bandwidth.

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